Enhancing the Thermal Stability of Majorana Fermions with Redundancy Using Dipoles in Optical Lattices

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Pairing between spinless fermions can generate Majorana fermion excitations that exhibit intriguing properties arising from nonlocal correlations. But, simple models indicate that nonlocal correlation between Majorana fermions becomes unstable at nonzero temperatures. We address this issue by showing that anisotropic interactions between dipolar fermions in optical lattices can be used to significantly enhance thermal stability. We construct a model of oriented dipolar fermions in a square optical lattice. We find that domains established by strong interactions exhibit enhanced correlation between Majorana fermions over large distances and long times even at finite temperatures, suitable for stable redundancy encoding of quantum information. Our approach can be generalized to a variety of configurations and other systems, such as quantum wire arrays.

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Introduction.—The wide variety of optical lattice geometries offers unprecedented tunability in manipulating quantum degenerate gases into complex quantum states [1]. Recent developments in the cooling of molecules (e.g., $^{40}K^{87}Rb$) [2] and magnetic atoms (e.g., ^{161}Dy) [3] imply that anisotropy in dipolar interactions will soon provide further opportunity to explore some of the most elusive yet compelling quantum states, entangled Majorana fermions (MFs).

Seminal lattice models demonstrate particlelike excitations that behave as MFs thanks to nonlocal symmetries [4,5]. They entangle with each other over large distances through string operator (SO) correlations. In simple models, SOs have straightforward definitions, e.g., fermion parity [4], with nontrivial consequences. They signal underlying topological order with fascinating properties that have motivated proposals for topologically protected qubits [5,6]. The crossing of SOs is responsible for unusual anyonic braid statistics [5,7]. And, SOs connecting these excitations also underlie theories of quantum state teleportation [8,9].

The zero-temperature properties of models hosting topological order set the stage for work connected to experiments. Kitaev's two-dimensional (2D) toric code Hamiltonian [5] motivated early proposals in optical lattices [10–12]. But, the 1D Kitaev chain model [4] is one of the simplest models supporting MF excitations. Anticipation of nonlocal MF properties in 1D led to experimental proposals and experiments in both optical lattices [13–15] and solids [4,16,17]. But, prospects for observing the nonlocal correlation of MF pairs over long times and distances hinge on the stability of SOs [7,18].

SOs in important lattice models are unstable at nonzero temperatures. For example, SOs in the 2D toric code model vanish at long times and distances because of thermal excitations [7,18–20]. Recent work also argues that MFs in lattice models of topological p-wave superconductors are sensitive to thermal fluctuations [21,22]. A general theorem [20] sets strict criteria for nonlocal correlations to remain resilient against thermal fluctuations. Fortunately, recent calculations indicate that topological phases can be enhanced through disorder [23] and proximity coupling [24,25] to a reservoir in topological superconducting wires [16]. There are also proposals to go beyond 1D wires to multichannel or 2D MF arrays [26].

We propose that dipolar interactions in optical lattices [27] offer a powerful tool to stabilize the SOs in MF models. We show that anisotropy in both the lattice and dipolar interactions electrostatically copies SOs to force excitations to form arrays of strings which we call *domains* in this work. We thus propose a robust mechanism, the formation of domains with redundant MF edges, as a route to stabilize MFs, akin to quantum error correction schemes using redundant qubits [28]. We pair two methods [quantum Monte Carlo (QMC) calculations and mean field theory] to solve a model of dipolar fermions to demonstrate that domain formation in electrostatically coupled Kitaev chains significantly enhances the stability of SOs. QMC calculations here are unbiased and show the thermal stability of domains, while our mean field theory (which agrees with QMC calculations within regimes of applicability) explicitly reveals MFs.

Model.—We first consider a Hubbard model of dipolar fermions in an $L \times L$ optical lattice and then discuss a specific parameter regime. In Fig. 1, fermions with dipolar moment $\vec{p}$ can hop between nearest neighbor (NN) sites. A large optical lattice depth along the y direction strongly suppresses hopping in the y direction. $V_x(\theta) = D^2(1 - 3\cos^2\theta)/r_0^3$ ($V_y = D^2/r_0^3$) is the x (y) component of the NN dipolar fermion interaction. Here, $D^2 \sim \vec{p}^2$ and r_0 is a lattice constant. We can tune θ so that the NN dipolar interaction is attractive along the x direction. We construct a Hubbard model capturing the above features:

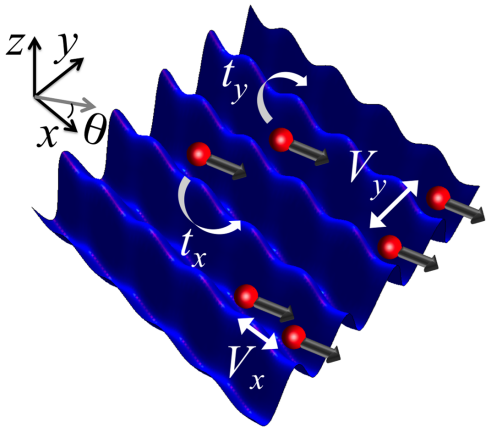

FIG. 1 (color online). Schematic of dipolar fermions (spheres) in a 2D optical lattice. Dipolar moments $\vec{p}$ (arrows on each sphere) align along an applied field, at an angle θ with the x axis.

$$H_D = -\sum_{i,j}(t_x a^\dagger_{i,j}a_{i+1,j} + t_y a^\dagger_{i,j}a_{i,j+1} + \text{H.c.}) + \sum_{i,j}[V_x(\theta)n_{i,j}n_{i+1,j} + V_y n_{i,j}n_{i,j+1} - \mu_0 n_{i,j}], \quad (1)$$

where we have an open (periodic) boundary condition in the x (y) directions. $a^\dagger_{i,j}$ creates a spinless fermion at the site (i, j), and $n_{i,j} = a^\dagger_{i,j}a_{i,j}$. t_x (t_y) is the hopping energy between NN sites in the x (y) direction. μ_0 is the chemical potential.

For a range of θ yielding $V_x < 0$, the ground state of Eq. (1) is stable and exhibits p-wave pairing. For $t_x = t_y$, functional renormalization group [29] and mean field theory [30] calculations show a BCS paired state for long-range dipolar interactions consistent with short-range interactions in Eq. (1) [21]. p-wave pairing between neighbors along x rows can be modeled by real-space attraction: $\exp(\mathbf{i}\Phi_{i,j})|\Delta|a^\dagger_{i+1,j}a^\dagger_{i,j} + \text{H.c.}$, where $\Phi_{i,j}$ and $|\Delta|$ are the phase and magnitude of the pairing field within an x row. But, for $t_y \ll t_x$, the system can be analyzed with Luttinger liquid theory to show that weakly coupled 1D dipolar systems also possess p-wave pairing order with algebraically decaying pairing correlations [31]. For $t_y \ll |\Delta|$, Josephson tunneling between paired states contributes an energy: $\sim -t_y^2\cos(\Phi_{i,j} - \Phi_{i,j+1})$, which aligns the phase of the pairing field between each x row $\Phi_{i,j} - \Phi_{i,j+1} \to 0$. Hereafter, we assume a uniform pairing field to motivate a thermally stable MF model. Increasing t_y should adiabatically connect the coupled 1D [31] and 2D square lattice limits [29,30].

Effective model.—We perform a mean field decoupling of the attractive dipolar interaction term in Eq. (1) to establish the centerpiece of our study [32]:

$$H_F = \sum_j H^j_K + V_y\sum_{i,j}\left(n_{i,j} - \frac{1}{2}\right)\left(n_{i,j+1} - \frac{1}{2}\right), \quad (2)$$

where the Hamiltonian for the jth Kitaev chain is $H^j_K = -t\sum_i(a^\dagger_{i,j} - a_{i,j})(a^\dagger_{i+1,j} + a_{i+1,j}) - \mu n_{i,j}$. At the Hartree-Fock level, the chemical potential renormalizes to $\mu = \mu_0 + 2\langle n_{i,j}\rangle|V_x(\theta)| - V_y/2$ and the hopping becomes $t = t_x - |V_x(\theta)|\langle a^\dagger_{i+1,j}a_{i,j}\rangle$, which is our energy unit. In Eq. (2), we tuned V_x to match the pairing term with the renormalized hopping by setting $t_x = |V_x(\theta)|\langle a^\dagger_{i+1,j}a^\dagger_{i,j} + a^\dagger_{i+1,j}a_{i,j}\rangle$. MFs can arise away from this particular point, which is guaranteed by the presence of a gap in the energy spectrum of H_F [33]. t_y is energetically negligible but is included as a second order effect by setting $\Phi_{i,j} = 0$. We work near half-filling $\langle n\rangle = 1/2$, i.e., $\mu = 0$.

Equation (2) describes an array of strongly interacting Kitaev chains, whose ground state is 2^L-fold degenerate [32], which is not explicit in Eq. (1). Our direct QMC simulations on Eq. (1) show the emergence of precisely the same set of degeneracies expected from Eq. (2) for the parameters given by the Hartree-Fock decoupling [32,34].

Mechanism for stabilizing MFs.—Equation (2) is a highly nontrivial many-body model. It maps onto an intractable quantum spin compass model [32,33]. Below, we argue that the interchain interactions stabilize correlation between edge y columns of MFs.

We use mean field theory to show that Eq. (2) reduces to a MF model [32]. Consider a pair of MF operators $c_{2i,j}$ and $c_{2i-1,j}$ for each site of the lattice (i, j), where $a^\dagger_{i,j} = (c_{2i-1,j} - \mathbf{i}c_{2i,j})/2$ [4]. We impose a mean field decoupling of the V_y term, using a two-site unit cell along the y direction. Each site of the unit cell corresponds to sublattice A or B. We thus have $H^\alpha_M = \mathbf{i}t\sum_i c_{2i,\alpha}c_{2i+1,\alpha} + (\mathbf{i}\tilde{\mu}_\alpha/2)\sum_i c_{2i-1,\alpha}c_{2i,\alpha}$, where $\alpha \in \{A, B\}$ denotes sublattice and the renormalized chemical potential $\tilde{\mu}_\alpha = \mu + \mathbf{i}V_y\langle c_{2i-1,\alpha}c_{2i,\alpha}\rangle$. Furthermore, we can show [32] that the ground state avoids strong V_y by setting $\langle c_{2i-1,\alpha}c_{2i,\alpha}\rangle = 0$ for $V_y > 4t$. This leads to two columns of localized MF states, one at each edge.

Solutions of H_M exhibit domains with MF edge states along y columns (Fig. 2) [32]. Note that the V_y term in Eq. (2) leads to a chemical potential staggered along y columns, which binds MFs along y but leaves them to propagate along x. An energy penalty $\sim V_y$ will result if only one row changes its parity. The inter-row interaction therefore increases the dimension of the MF edge state (from a point particle to a y column) to establish the mechanism for enhancing the stability of the nonlocal MF state against thermal fluctuations. The entire ground state can thus be regarded as a redundantly encoded qubit of several MFs. Along these lines, mean field theory suggests the following Gutzwiller projected wave function: $\prod^L_{i,j=1}(1 - n_{i,j}n_{i,j+1})\phi^j_{\text{BCS}}$, where ϕ^j_{BCS} is the BCS wave function hosting MFs in the jth x row.

Thermally stable nonlocal correlation implies that y columns of MF pairs at $i = 1$ and $i = L$ host real dipoles in a superposition that remains robust against thermal excitations. To establish robustness, we note that the

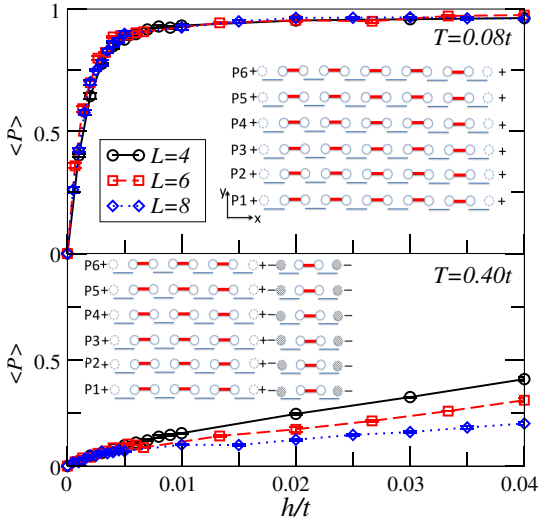

FIG. 2 (color online). The thermal expectation value of SOs from QMC calculations as a function of an applied global field for several system sizes for $V_y = 4.8t$ and $\mu = 0$. The top (bottom) panel shows data for a characteristic low (high) temperature. The insets show schematic examples of a MF domain that breaks up into two MF domains at high temperatures. The +'s in the figures is fermion parity for the entire chain, and each chain has the same parity for the one configuration drawn. Empty dashed circles denote empty MF edge states; hatched circles denote MF edge states occupied by one particle per row.

Hilbert space of Eq. (2) possesses a spectral gap ΔE above a degenerate manifold of states for the parameters we consider here [33]. But, the entropy gain S in the free energy cost to create excitations $\Delta E - TS$ can overwhelm the energy gap, depending on the effective dimensionality of excitations. Strong interactions $V_y > 4t$ require the creation of entire domains (with a perimeter $\sim L$, $\Delta E \sim L$, and $S \sim L$) to destroy nonlocal correlations, as opposed to $\Delta E \sim \mathcal{O}(1)$ and $S \sim \log L$ for $V_y < 4t$. Favorable entropy scaling implies that nonlocal correlation between MF y columns in 2D is much more thermodynamically stable than between pairs of individual MFs in 1D.

QMC test of thermal stability.—We test the robustness of SOs of MFs with QMC simulations [35] on Eq. (2) [32]. The nonlocal correlation between edge states at $i = 1$ and $i = L$ is captured by a set of L SOs that stretch across each x row: $P_j \equiv \prod_{i=1}^{L}(1 - 2n_{i,j}) = (-1)^{\sum_i n_{i,j}}$, where $j = 1, 2, \ldots, L$ along y. P_j is equivalent to the fermion parity for the jth row.

The expectation values of the SOs P_j act as order parameters. Unique values $\langle P_j \rangle = \pm 1$ can be used to define each sector and therefore indicate stability in the nonlocal correlations between MFs. But, $\langle P \rangle = 0$ indicates that thermal excitations destroy any distinction between sectors. We compute $\langle P_j \rangle$ to show spontaneous breaking of these discrete symmetries for $V_y > 4t$ even at nonzero temperatures. To detect such a symmetry breaking, we perturb the above spinless fermion model with a weak global field: $H = H_F - \tilde{h}\sum_{j=1}^{L} P_j$. The global field $P = L^{-1}\sum_{j=1}^{L} P_j$ imposes a splitting between the otherwise degenerate states. We define $\tilde{h} = hL$ to ensure that the perturbing term imposes a nonzero energy splitting per particle h between degenerate sectors even in the limit $L \to \infty$. $h > 0$ favors $\langle P \rangle = 1$.

We first compute $\langle P \rangle$ in the limit $V_y < 4t$ using QMC calculations. For $V_y = 3.2t$, we find $\langle P \rangle \to 0$ with increasing L. This indicates that the SOs in 1D x rows alone are extremely sensitive to thermal fluctuations, as expected from the entropy argument above, even with $\Phi_{i,j}$ held constant. Our calculations are time independent. One may find $|\langle P \rangle| > 0$ at short times.

We now calculate $\langle P \rangle$ in the strongly interacting case $V_y = 4.8t$, where we expect arrays of strings to form stable domains. Figure 2 shows $\langle P \rangle$ at low and high temperatures. At high T, the bottom panel shows that a large value of h is needed to stabilize the SOs. But, at low T (top panel), we find that very small fields tend to force all x rows to spontaneously occupy the lowest energy state in the limit $h \to 0$, which indicates that y columns of MFs located at $i = 1$ and $i = L$ can be prepared in a long-lasting entangled state stretching over large distances even at finite temperatures.

Thermal stability of domains.—The arrays of SOs defining domains are stable at low temperatures but eventually break up at large T. To find the critical temperature for domain formation, we define a string-string order parameter that captures the ordering strength along the y direction: $\langle O \rangle \equiv L^{-2}\sum_{j,j'=1}^{L}\langle P_j P_{j'} \rangle$. The operator O is similar to the static structure factor $S_{k_y} \propto \sum_{j,j'=1}^{L} \exp[-\mathbf{i}k_y(j - j')]\langle n_j n_{j'} \rangle$, but with the replacement $n_j n_{j'} \to P_j P_{j'}$ and with wave vector $k_y = 0$.

We look for long-range order in the susceptibility of O: $\chi_O = L^2(\langle O^2 \rangle - \langle O \rangle^2)/T$. A peak in χ_O versus T indicates the critical temperature T_c at which the large domain breaks up along the y direction. For $V_y < 4t$, we find no peaks in our simulations and therefore no domain formation for weakly interacting chains, i.e., $T_c = 0$.

We observe domain formation in χ_O for $V_y > 4t$. The top panel of Fig. 3 shows χ_O as a function of temperature for $V_y = 4.8t$. Above T_c, the y columns of MFs are no longer ordered. The bottom panel extracts T_c in the thermodynamic limit, yielding $T_c = 0.275(4)t$. Our results agree with studies on the quantum compass model, showing a thermal phase transition in the universality class of the 2D Ising model [36].

The robustness of the ground state degeneracy also reveals the stability of the SOs. We denote each ground state energy sector by $E(P_1, P_2, \ldots)$. We found that this degeneracy was not lifted with a weak staggered chemical potential, interchain hopping, or a uniform chemical potential shift [34]. We present representative results for the uniform chemical potential shift. Figure 4 shows the energy

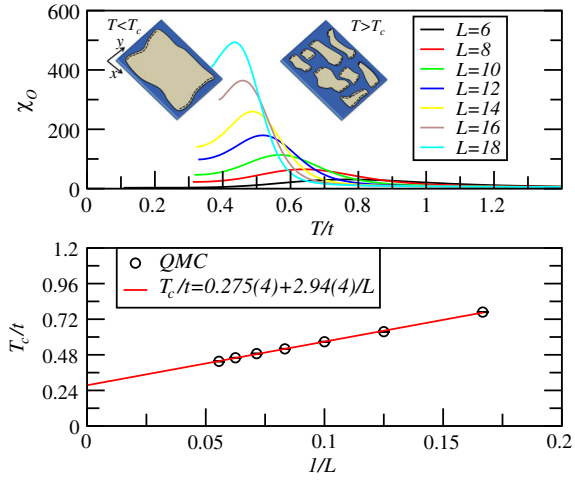

FIG. 3 (color online). Top: The susceptibility of the string-string correlation function O from QMC simulations for different L's at $V_y = 4.8t$ and $\mu = 0$. The SOs tend to order along the y direction for $T < T_c$. The inset shows a schematic of an ordered domain with MFs forming columns at the ends (dashed lines). The domains shrink for $T > T_c$. Bottom: T_c extrapolated to $L \to \infty$. The solid line is a linear chi-squared fit.

splitting per particle of two different sectors of the P_j operator: $\delta E \equiv E(-1, -1, \ldots) - E(1, 1, \ldots)$, as a function of μ. The flat portion for $\mu/t \ll 1$ indicates a robust degeneracy. Above $\mu \approx 1.5t$, the energy splitting acquires a size dependence, as expected for $\mu > \Delta E$. Inset (a) shows that the particle density has weak linear dependence for $\mu/t \ll 1$, which is also captured by the mean field theory.

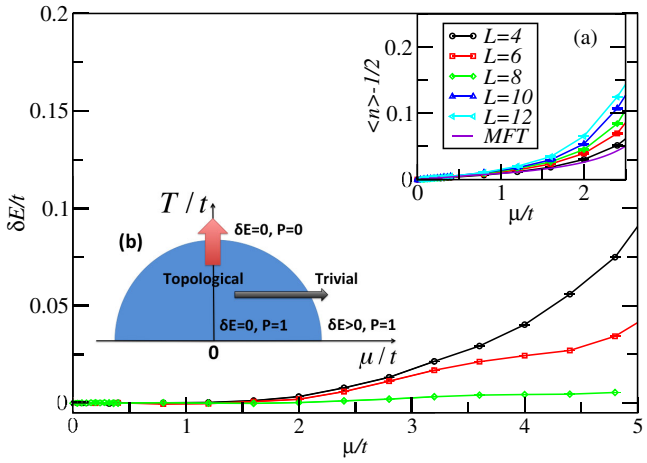

FIG. 4 (color online). The main panel plots the energy splitting between two sectors defined by $P_j = \pm 1$ for all x rows as a function of chemical potential for Eq. (2) at $T = 0.16t$ and $V_y = 4.8t$. Inset (a) shows a weak linear increase in density with increasing μ inside the topological phase ($\mu \lesssim 1.5t$). Inset (b) shows a schematic phase diagram established by the lifting of the degeneracy; see the horizontal arrow. The vertical arrow indicates the thermal phase transition explored in Fig. 3. MFT denotes the mean field theory result.

Our results are consistent with the formation of a thermally robust topological phase, shown in inset (b) of Fig. 4.

Detection in optical lattices.—Domain formation can be observed directly in time-of-flight measurements. Noise correlations between shots of individual time-of-flight images relate to S_k [37]. In the topological phase, we anticipate the formation of lines, rather than peaks, in noise correlations because the V_y term correlates the density along just the y direction for $T < T_c$. Observations of these lines should therefore allow identification of T_c.

Correlation between MFs could be demonstrated through nonlocal measures similar to those proposed in quantum wires [9]. Local spectroscopic probes [13,15] applied at each domain edge could be adapted to detect the response of one domain edge when dipoles are added to alternating Kitaev chains on the opposite edge. The particle number parity in the opposite edge should respond with signatures of nonlocal correlations in dynamics [9]. Recent experiments using high resolution spectroscopy to measure particle number parity [38] and SOs [39] could be used to explicitly measure response.

Fluctuations in pairing.—We connected a model of oriented fermionic dipoles [Eq. (1)] to a pairing model [Eq. (2)]. The pairing model itself demonstrates significantly enhanced stability of the MF state via domain formation at $T > 0$. But, our specific implementation still allows fluctuations of the pairing field between x rows. Fortunately, the long-range dipolar interaction has been found to enhance the stability of p-wave superfluidity [30].

Coherent reservoirs can further suppress pairing field fluctuations via the proximity effect [14,15,25]. We can show that an optical lattice geometry allowing proximity coupling is possible [32]. We note, however, that excitations in the system may couple to those in the reservoir [24].

Conclusion.—We considered an effective model of oriented dipolar fermions in a 2D lattice that allows hopping along directions where the dipoles attract but suppresses hopping along directions where dipoles repel. In the p-wave superfluid regime, we model the system with repulsive Kitaev chains. Each chain experiences a self-consistently renormalized chemical potential due to its neighbor to impose an energy penalty for excitations. This energy penalty is the mechanism behind MF domain formation and therefore enhances correlation between columns of MFs along each domain edge. Unbiased QMC calculations confirm that string operators defining nonlocal MF states remain robust to thermal fluctuations.

Our approach generalizes to a variety of lattice geometries and even other models with MFs, provided they take a similar form: $\sum_a H_M^a + \sum_{a,b} V_{\text{int}}^{a,b}$, where H_M^a defines a model with MFs, $V_{\text{int}}^{a,b}$ creates domains with diagonal interactions between models a and b, and $V_{\text{int}}^{a,b}$ does not commute with H_M^a [20]. This class of Hamiltonians also applies to Coulomb coupling in MF models of quantum wire arrays or quasi-1D tubes containing topological superconductors.

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Note added.—Recently, we became aware of work on similar nonlocal order parameters [40].

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