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POSSIBLE NEW PHASES OF COMPOSITE FERMIONS

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When the effective filling factor of composite fermions is an integer, the residual interaction between them can often be neglected because the ground state of the non-interacting model is unique and incompressible. However, at non-integer composite fermion (CF) filling factors the ground state of composite fermions is enormously degenerate if the interaction between them is neglected, and consideration of the inter composite fermion interaction is necessary for determining the true ground state. In this article, we summarize certain results regarding what new states the inter composite fermion interaction can possibly produce. More details can be found in Refs. [11] and [12].

1 Introduction

A hypothetical system of *non-interacting* electrons in two dimensions, exposed to a high magnetic field, would only show the integral quantum Hall effect, because gaps appear only at integral filling factors, due to Landau level quantization of the kinetic energy. At non-integral filling factors, the ground state of non-interacting electrons is extremely highly degenerate, and interactions play a crucial role in lifting the degeneracy and determining the true ground state.

The essential consequence of the interaction is to produce composite fermions, electrons bound to and even number of quantum vortices of the many body wave function.[1,2] The simplest approximation, in which the composite fermions are assumed to be non-interacting, has been quite successful in explaining the phenomenology. The integral quantum Hall effect of composite fermions carrying $2p$ vortices (called ^{2p}CFs) at effective filling $\nu^*=n$ corresponds to the fractional quantum Hall effect (FQHE) (see Ref. [3]) of electrons at $\nu = n/(2pn\pm1)$, which are the most prominently observed fractions. At $\nu=1/2p$, the $n\to\infty$ limit of the above sequences, a Fermi Sea of composite fermions is obtained,[4] which, in the absence of CF-CF interactions, has no gap. This explains the lack of FQHE at the simplest even denominator fractions; the existence of the CF Fermi sea has been confirmed in several experiments.[5]

However, at $\nu^*\neq n$, the ground state for the model system of non-interacting composite fermions is highly degenerate, and it is crucial to take account of the CF-CF interaction to determine the nature of the true ground state. Would the composite fermions capture more vortices to become higher order composite fermions to show more FQHE, or would they form some new state?

2 Variational States

We have explored this question in the context of filling factors $\nu=(2n+1)/[4(n+1)]$, which correspond to $\nu^*=n+1/2$ of composite fermions. The n filled Landau levels of composite fermions are treated as inert, and the problem is mapped into fermions at half filling.

We proceed in a variational approach, considering the following plausible states:

(i) ^{4}CF Fermi sea: The ^{2}CFs capture two additional vortices to convert into ^{4}CFs, which experience no magnetic field and form a Fermi sea. This state is well described by the wave function

$$\Psi_{FS}=P_{LLL}\Phi_1^2\,\Phi_\infty \tag{1}$$

where Φ_∞ is the Fermi sea wave function at zero magnetic field, Φ_1 is the wave function of the lowest filled Landau level, and P_{LLL} is the lowest Landau level projection operator. The base particles in Φ_∞ are ^{2}CFs, so Ψ_{FS} is a Fermi sea of ^{4}CFs.

(ii) ^{4}CF paired state: The ^{2}CFs capture two additional vortices to convert into ^{4}CFs, which pair up. A gap opens up due to pairing, which results in a FQHE. This mechanism appears to be relevant for the FQHE at $\nu=5/2$.(see Ref.[6]) A satisfactory approximation for this state is the Pfaffian wave function[7]

$$\Psi_{PS}=\Phi_1^2\,Pf[M] \tag{2}$$

where $Pf[M]$ is the Pfaffian of the NxN antisymmetric matrix M with components $M_{jk}=(z_j-z_k)^{-1}$, and $z=x-iy$ denotes the position of a particle in the plane. $Pf[M]$ is a real space BCS wave function, so Ψ_{PS} describes a paired state of composite fermions.

(iii) ^{2}CF unidirectional charge density wave: The ^{2}CFs phase separate into stripes of alternating FQHE states. A Hartree-Fock wave function for this state can be written and its energy evaluated.[8]

(iv) ^{2}CF two-dimensional charge density wave: The ^{2}CFs form what is called a bubble crystal.[8] Again, the energy of this state can be estimated in the Hartree-Fock approximation.

The last two states are not unique, in the sense that the period of the stripe or the crystal (which depends on the number of particles in each bubble) is variable, and the lowest energy must be determined in each case variationally. The methods for obtaining the energies of the paired and Fermi sea states as well as the charge density wave states have been described in detail in the literature.[8-10]

It ought to be noted that the above states are fantastically complicated, correlated states when viewed in terms of electrons. For example, in (ii): first all electrons at $\nu=(2n+1)/4(n+1)$ capture vortices to become ^{2}CFs at $\nu^*=n+1/2$; those in the topmost half filled ^{2}CF Landau level capture two additional vortices to transform into ^{4}CFs that see no magnetic field; these would normally form a ^{4}CF Fermi sea, which is here unstable to pairing due to a weak residual interaction between the ^{4}CFs; a gap opens up due to pairing and the FQHE is produced.

3 CF-CF Interaction

In order to determine the lowest energy state, we need a model for the effective interaction between the composite fermions in the topmost half filled CF Landau level. Given the strongly correlated nature of the problem, the interaction is complicated and is expected to contain two, three, and higher body terms. In order to make progress, we will neglect all but the two-body term, which we will determine by placing only two CFs in the nth CF Landau level, while filling the lower CF Landau levels completely. The validity of this approximation is discussed in Refs.[11,12] The wave function for this state is uniquely given by the composite fermion theory as a function of the relative angular momentum, from which one can work backward to obtain a real space interaction. The detailed method has been explained in the literature.[10,11,13]

4 Results

We consider two situations. First, we assume that the external magnetic field is sufficiently high that the system is fully spin polarized. Fig. 1 shows the energies of the paired CF state as well as the CF Fermi sea as a function of $1/N$, where N is the number of composite fermions in the topmost CF Landau level. The thermodynamic limit is obtained by linear extrapolation. The energies of the stripe and bubble phases are found in the Hartree-Fock approximation, which directly gives the thermodynamic limit.

A comparison of the energies shows that stripes have the lowest energy at total fillings 3/8, 5/12, and 7/16, which lie between 1/3 and 2/5, 2/5 and 3/7, and 3/7 and 4/9, respectively. The anisotropic transport observed in higher electronic Landau levels has been interpreted in terms of the formation of stripes.[14] A similar anisotropy between fractions will indicate the formation of stripes alternating between n and n+1 filled Landau levels of composite fermions.

The conditions for the observation of CF stripes in the FQHE regime are obviously more stringent than those for the observation of electron stripes in higher Landau levels, just as the conditions for the observation of the fractional quantum Hall effect are than the integral quantum Hall effect. The effective interaction between the composite fermions is approximately an order of magnitude smaller than that between electrons in higher Landau levels, indicating that the temperatures at which the CF stripes become observable might also be reduced similarly. The

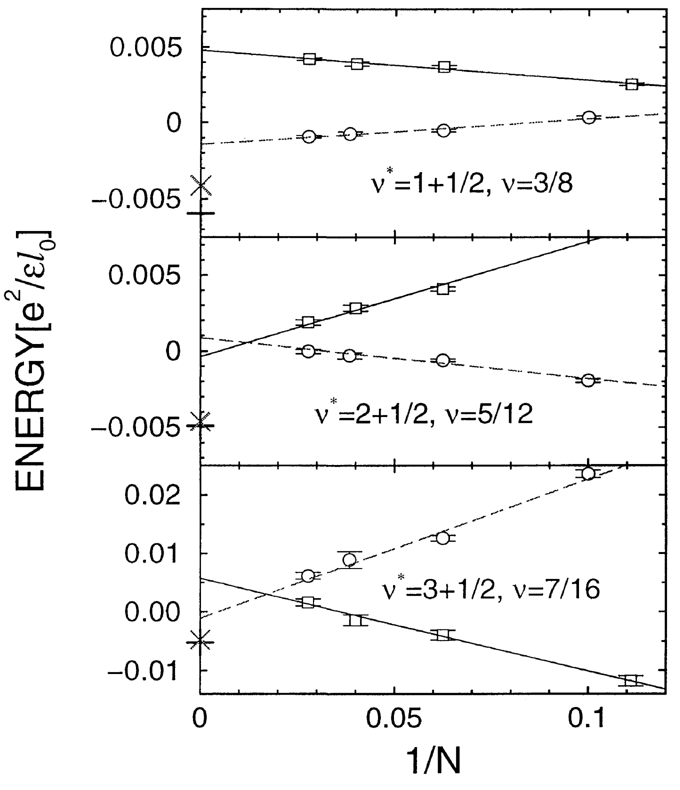

Figure 1: The energy per particle for the CF Fermi sea (squares) and the CF paired state (circles) as a function of N, the number of composite fermions in the (n+1)st spin up CF Landau level. The thermodynamic energies are also indicated for the CF stripe and bubble phases by dashes and crosses on the y-axis. All energies are measured relative to the uncorrelated uniform density state.

period of the CF stripes is also much larger: 10, 28, and 34 in units of the magnetic length for ν=3/8, 5/12, and 7/16, respectively. The reason is that the CF stripes involve density oscillations of much smaller amplitude than the electron stripes in the integral quantum Hall regime, as the neighboring FQHE states have very similar densities.

Next we consider the situation when the composite fermions in the topmost CF Landau level have spin opposite to composite fermions in the fully occupied CF Landau level. This state would be irrelevant in the $B \to \infty$ limit, but could occur for typical or small magnetic fields. The calculation proceeds as before, but with a different effective interaction, appropriate for spin reversed composite fermions.

Figure 2 shows how the thermodynamic energies of the Fermi sea, paired state, and the stripe state vary at ν=3/8 as the first pseudopotential[15] of the effective interaction, V^{eff}, is changed. V^{eff} is the effective interaction obtained from the microscopic method described above. It is seen that when the short range part is enhanced the Fermi sea wins; when it is significantly reduced the stripe phase has the lowest energy; but there is a range of parameters where the paired state is the relevant state. Further calculations[12] have confirmed this result for the above model, but show that the paired state is rather delicate, with an extremely small excitation gap.

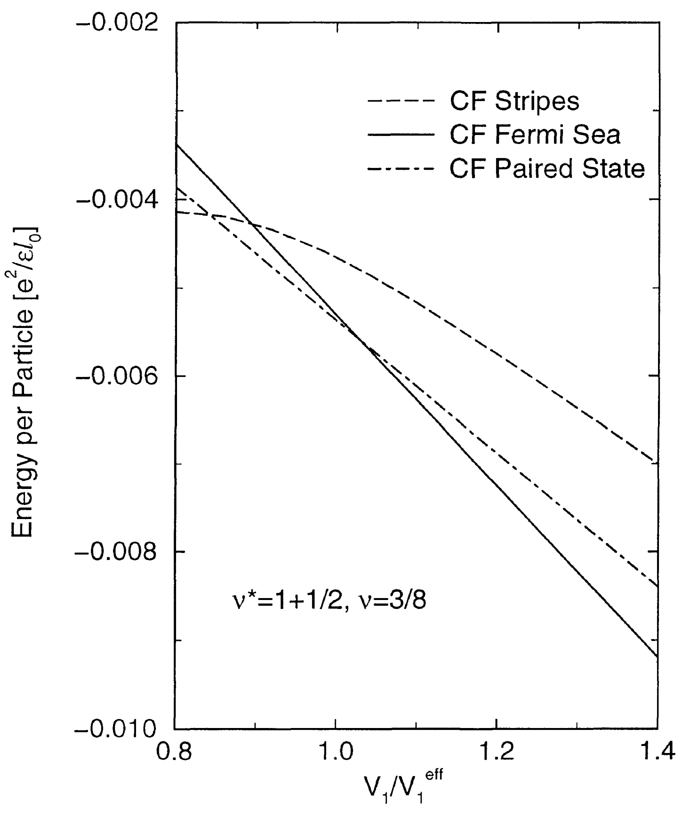

Figure 2: The thermodynamic energies of the CF stripe (dashed line), CF Fermi sea (solid line), and CF paired (dot-dashed line) states as a function of the first pseudopotential for the interaction modeling one full spin up CF Landau level and a half filled spin down CF Landau level (ν*=1+1/2; ν=3/8). The energies are measured relative to the uncorrelated uniform density state.

In summary, out studies suggest the possibility of new structure *between* the principal FQHE sequences, due to either stripe formation or pairing of composite fermions. The energy scale associated with these phases is estimated to be rather low, approximately an order of magnitude below the analogous states in higher Landau levels. This work was supported in part by the National Science Foundation under grants DMR-9986806 and DGE-9987589.

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