Stripe Formation in the Fractional Quantum Hall Regime

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An understanding of the physics of the half-filled lowest Landau level has been achieved in terms of a Fermi sea of composite fermions, but the nature of the state at other even-denominator fractions has remained unclear. We investigate theoretically Landau level fillings of the form $\nu = (2n + 1)/(4n + 4)$, which correspond to composite-fermion fillings $\nu^* = n + 1/2$. By considering various plausible candidate states with complete spin polarization, we rule out the composite-fermion Fermi sea and paired composite-fermion state at these filling factors, and predict that the system phase separates into stripes of n and $n + 1$ filled Landau levels of composite fermions.

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The Coulomb interaction between electrons in two dimensions confined to the lowest Landau level (LLL) expresses itself most strongly through the binding of an even number of quantum mechanical vortices on each electron and thereby creating composite fermions (CFs) [1,2]. The residual interaction between composite fermions is weak and unimportant in many situations, in the sense that it does not alter the nature of the state in a qualitative manner. For example, at the odd-denominator fractions $\nu = n/(2pn \pm 1)$, which correspond to integral fillings $\nu^* = n$ of composite fermions carrying $2p$ vortices, the intercomposite fermion (inter-CF) interaction can often be neglected due to the presence of the effective cyclotron gap. While the model of free composite fermions is adequate for understanding the basic phenomenology of the fractional quantum Hall effect (FQHE) [3], there has been much interest recently in the more subtle physics arising from the weak inter-CF interaction.

The state at $\nu = 1/2p$, obtained in the $n \to \infty$ limit of the $\nu = n/(2pn \pm 1)$ sequence, is well described as the Fermi sea of composite fermions carrying $2p$ vortices [4,5], called ^{2p}CFs. However, our understanding of other even-denominator fractions is less satisfactory. We will focus in this paper on even-denominator fractions of the form $\nu = (2n + 1)/(4n + 4)$, which correspond to composite fermion filling factors $\nu^* = n + 1/2$, according to the relation $\nu = \nu^*/(2\nu^* + 1)$. At these filling factors, the topmost CF-Landau level (CF-LL) is only partially occupied, and it is crucial to take account of the inter-CF interaction, without which the ground state would have an enormous degeneracy. We ask what finer structure the weak interaction produces. A fully spin polarized state will be assumed as appropriate for sufficiently high magnetic fields. The conclusions will also apply to $\nu = (2n + 3)/(4n + 4)$ due to particle hole symmetry in the lowest LL.

Several interesting states have been considered at *electron* filling factors $\nu = n + 1/2$, which will serve as paradigms for the discussion below. In the lowest Landau level ($\nu = 1/2$), electrons transform into composite fermions which condense into a ^{2}CF Fermi sea [4,5]. In the second LL ($\nu = 2 + 1/2$) it appears that electrons turn into composite fermions, which form a BCS-like state to produce FQHE [6–9]. In yet higher Landau levels, electrons do not capture vortices but are instead believed to exhibit a stripe phase [10–14].

Which state actually occurs depends on the interaction matrix elements, and it is therefore important to have a good model for the inter-CF interaction, $V^{\rm CF}(r)$. We proceed as follows: In order to treat the state at $\nu^* = n + 1/2$, we start with the state with $\nu^* = n$ filled Landau levels of composite fermions and add two additional composite fermions in the lowest empty CF-Landau level. Following the standard procedure for writing the wave functions for composite fermions [2], the wave function for this state is given by $\mathcal{P}_{\rm LLL}\Phi_1^2\Phi_n^{++m}$, where Φ_1 is the wave function of one filled LL, Φ_n^{++m} is the wave function of the *electron* state in which n LLs are fully occupied, and the $(n + 1)$st LL contains two electrons in a relative angular momentum m state, and $\mathcal{P}_{\rm LLL}$ is the lowest LL projection operator. The explicit form for the general wave functions of this type is given in the literature, as also are detailed tests of their validity [15]; the calculation of energy requires the evaluation of multidimensional integrals which is accomplished by the Monte Carlo method. (The spherical geometry [16] is used in our calculations.) This provides the pseudopotentials [16] $V_m^{\rm CF}$, which completely specify the interaction between two composite fermions in the $(n + 1)$st CF-LL. Similar studies have been done previously [17], except that here $V_m^{\rm CF}$ are evaluated for fairly large systems, and are closer to the thermodynamic limit. We then construct a real space interaction between composite fermions; for convenience, we map the problem of composite fermions in any arbitrary CF-LL into a problem of fermions in the lowest LL. There is no unique prescription for this, because many real space interactions produce the same pseudopotentials, but we find it convenient to use a potential of the form [9]

$$V^{\rm CF}(r) = \left(\sum_j c_j r^{2j} e^{-r^2} + \frac{(2n + 1)^{-5/2}}{r}\right)\left[\frac{e^2}{\epsilon l_0}\right]. \quad (1)$$

The last term gives the Coulomb interaction between two particles of appropriate fractional charge. The distance r is measured in units of the *effective* magnetic length ℓ, but the energies are measured in units of $e^2/\epsilon l_0$, where l_0 is the magnetic length at the actual electron filling factor ν, and ϵ is the dielectric constant of the background material. We fix the first few parameters c_j by requiring that $V^{\rm CF}(r)$ produce the first 5 to 6 odd pseudopotentials exactly. A comparison between $V_m^{\rm CF}$ and the pseudopotentials of $V^{\rm CF}(r)$ (Fig. 1) shows that $V^{\rm CF}(r)$ is a good approximation for all distances. We have thus mapped the problem of N composite fermions in the $(n+1)$st LL into that of N fermions at an effective filling in the lowest LL interacting with an effective potential. Only the composite fermions in the topmost half-filled CF-LLs will be considered explicitly; the completely occupied CF-LLs appear only through their role in determining the inter-CF interaction. We note that the interaction between composite fermions is markedly different from that between electrons in the corresponding higher LLs. In the second CF-LL, the interaction is most strongly repulsive in the $m = 3$ channel, and in higher CF-LLs, the interaction is actually attractive, with the lowest energy in the $m = 1$ channel.

Our conclusions below will be subject to two assumptions. (i) We assume that mixing with higher CF-LLs can be neglected, i.e., the inter-CF interaction is weak compared to the effective CF-cyclotron energy. There is evidence that this is an excellent approximation; the states containing several composite fermions are accurately described without considering mixing between CF-LLs [18]. (ii) We further assume that the interaction energy of many composite fermions in the $(n+1)$st CF-LL is well approximated by a sum of two-body terms. An examination of configurations containing three composite fermions in the second CF-LL provides some justification for this assumption [17,19], but further work will be required to ascertain the extent of its validity.

The first state that we look at is the Fermi sea, in which the ^{2}CFs capture two additional vortices to become ^{4}CFs, which then form a Fermi sea. (The composite fermions in the lower, fully occupied CF-LLs remain ^{2}CFs; this state thus contains an admixture of two different flavors of composite fermions.) The wave function of the Fermi sea is

$$\Psi_{\rm FS} = \mathcal{P}_{\rm LLL}\Phi_1^2\Phi_\infty\,. \tag{2}$$

The thermodynamic limit for the energy of the ^{4}CF sea is obtained by an extrapolation of finite system results, as shown in Fig. 2. Since we are interested in comparing energies obtained by different methods, it is important to carefully define the total energy; we will consistently take the same form for the electron-electron, electron-background, and background-background interactions in all our calculations. All energies are quoted relative to the energy of an uncorrelated uniform state explained below.

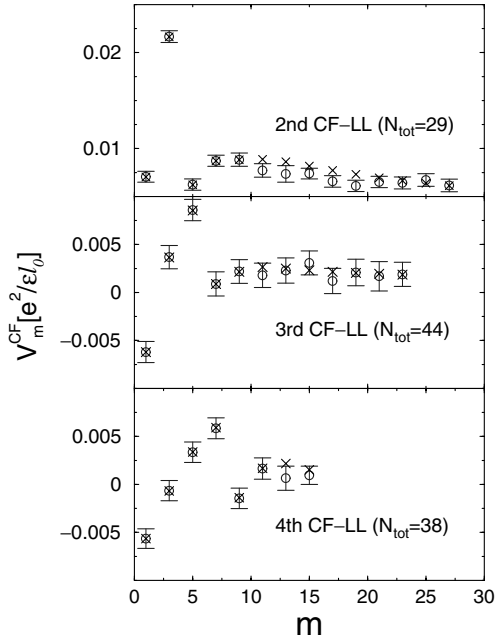

FIG. 1. The Haldane pseudopotentials for the intercomposite fermion interaction in the second, third, and fourth CF Landau levels, calculated from the microscopic wave functions (circles). The error bars indicate the statistical error from the Monte Carlo sampling. The crosses are the pseudopotentials for the model interaction $V^{\rm CF}(r)$ explained in the text. $N_{\rm tot}$ is the total number of particles.

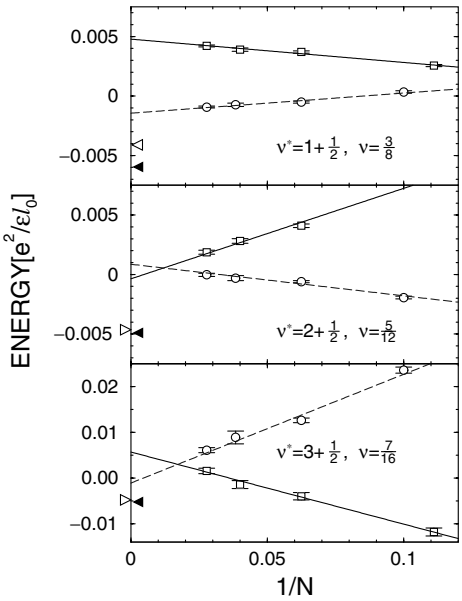

FIG. 2. The energy per particle for the CF Fermi sea (squares) and the CF paired state (circles) as a function of N, the number of composite fermions in the $(n+1)$st CF-LL. The thermodynamic energies are also indicated for the CF stripe and bubble phases by filled and empty arrows on the y axis. All energies are measured relative to the uncorrelated uniform density state, explained in the text.

We do not expect the ground state to be the ^{4}CF sea because the stability of the Fermi sea requires a strong short range repulsion, which is not the case with composite fermions in higher CF-LLs [20]. In fact, in the third and higher CF-LLs, the interaction between the composite fermions is attractive, which might suggest pairing of composite fermions. The paired state of composite fermions is represented by the Pfaffian wave function of Moore and Read [8]

$$\Psi_{\rm Pf} = \Phi_1^2 \, {\rm Pf}[M], \tag{3}$$

where ${\rm Pf}[M]$ is the Pfaffian of the $N \times N$ antisymmetric matrix M with components $M_{ij} = (u_i v_j - v_i u_j)^{-1}$, where $u_j \equiv \cos(\theta_j/2)\exp(-i\phi_j/2)$ and $v_j \equiv \sin(\theta_j/2)\exp(i\phi_j/2)$. ${\rm Pf}[M]$ is a real space BCS wave function, so $\Psi_{\rm Pf}$ describes a paired state of composite fermions. Again, since our base particle is a ^{2}CF, $\Psi_{\rm Pf}$ contains pairing of ^{4}CFs. The energy of this state, given in Fig. 2, beats the Fermi sea at $\nu = 3/8$ and $7/16$, raising the intriguing possibility of a FQHE, induced by pairing, at certain even-denominator fractions in the lowest Landau level.

Past studies [21] have shown that certain candidate FQHE states can be eliminated by looking at the "excitations." We consider the density-wave mode of the Pfaffian wave function in the single-mode approximation [22], described by the wave function $\mathcal{P}_{\rm LLL}\rho_k\Psi_{\rm Pf}$, where ρ_k is the density operator at wave vector k. We calculate the energy of this mode following Ref. [22], with the help of the pair correlation function of $\Psi_{\rm Pf}$ [23]. The energy, shown in Fig. 3 for $\nu = 3/8$ as a function of the wave vector, goes below the energy of the Pfaffian wave function, indicating that the paired state is unstable.

We have also carried out [19] exact diagonalization at the flux values corresponding to the Pfaffian wave function in the spherical geometry, and found that the ground state does not have orbital angular momentum $L = 0$, i.e., is not a uniform density state. The fact that the instability occurs at nonzero wave vectors in Fig. 3 also hints that the true ground state may not be a translationally invariant liquid.

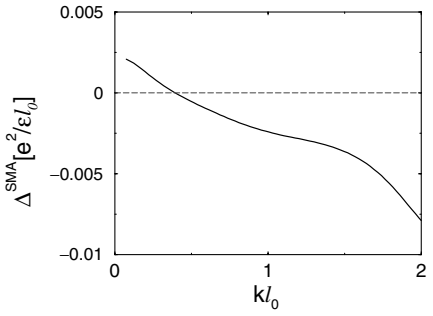

FIG. 3. The excitation energy of the single-mode-approximation (SMA) excitation for the Pfaffian wave function (solid line) at $\nu = 3/8$.

Besides pairing, another possible consequence of an attractive interaction is phase separation. Because of the long range Coulomb interaction, the phase separation is likely to manifest itself here through the formation of stripes or some other type of charge-density wave state. We calculate the Hartree-Fock energy of the stripe state of ^{2}CFs following the method of Koulakov *et al.* [10]. The interaction Hamiltonian is given by

$$\hat{V} = \frac{(2\pi)^3}{2L_xL_y}\sum_{\mathbf{q}} \tilde{V}^{\rm CF}(\mathbf{q})\rho(-\mathbf{q})\rho(\mathbf{q}), \tag{4}$$

where $\rho(\mathbf{q})$ is the density operator and $\tilde{V}^{\rm CF}(\mathbf{q})$ is the Fourier transform of $V^{\rm CF}(r)$. Subsequent to a Hartree-Fock decomposition, the expectation value of the interaction energy can be written as

$$\langle\hat{V}\rangle = \frac{(2\pi)^3}{2L_xL_y}\sum_{\mathbf{q}} \tilde{U}_{\rm HF}(q)\Delta(-\mathbf{q})\Delta(\mathbf{q}), \tag{5}$$

where $\Delta(\mathbf{q}) \equiv \frac{1}{2\pi}\sum_k e^{-ikq_x\ell^2}\langle a^\dagger_{k_+}a_{k_-}\rangle$, $\tilde{U}_{\rm HF}(q) = \tilde{U}(q) - \ell^2 U(q\ell^2)$, $\tilde{U}(q) \equiv V^{\rm CF}(q)e^{-\frac{1}{2}q^2\ell^2}$, and $k_\pm = k \mp q_y/2$. The $\mathbf{q} = 0$ term, which corresponds to the uniform (uncorrelated) state, is treated separately: the direct part is canceled by the background, and the exchange part is taken as the reference energy, given by

$$E_0 = -N\overline{\nu}\,\frac{U(0)}{2}, \tag{6}$$

where $\overline{\nu} = 1/2$. The contribution coming from nonzero values of $\mathbf{q}$ is called the coherence energy, $E_{\rm coh}$.

The stripe phase with period Λ corresponds to the choice

$$\Delta(x,y) = \frac{1}{2\pi\ell^2}\sum_q \frac{2\sin(\frac{q\Lambda\overline{\nu}}{2})}{\Lambda q}\, e^{iqx}, \tag{7}$$

where $q = \frac{2j\pi}{\Lambda}$, with $j = \pm1, \pm2, \ldots$. This gives

$$E_{\rm coh} = \frac{1}{2\overline{\nu}\ell^2}\sum_q \tilde{U}_{\rm HF}(q)\left(\frac{2\sin(\frac{q\Lambda\overline{\nu}}{2})}{\Lambda q}\right)^2. \tag{8}$$

We compute it as a function of Λ. The lowest energy, shown in Fig. 2, is obtained at $\Lambda/l_0 = 10$, 28, and 34 for $\nu = 3/8$, $5/12$, and $7/16$, respectively. The period is rather large compared to that for the electron stripes in higher LLs (for which Λ/l_0 is of order unity), which is not surprising because the interaction between composite fermions is rather weak, and also the difference between the densities of the FQHE states on either side is quite small.

We next consider the Wigner crystal of "bubbles" [10], with each bubble containing in general several electrons. For a hexagonal lattice with lattice constant Λ_b, the radius of a bubble is $R = \sqrt{\sqrt{3}\,\overline{\nu}/2\pi}\,\Lambda_b$ and

$$\Delta(\mathbf{r}) = (2/\sqrt{3}\,\Lambda_b^2)\sum_{\mathbf{q}} \frac{R}{\ell^2 q}\, J_1(qR)e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{9}$$

where $\mathbf{q} = m_1\mathbf{e}_1 + m_2\mathbf{e}_2$ with $\mathbf{e}_1 = (4\pi/\sqrt{3}\,\Lambda_b)\hat{y}$, $\mathbf{e}_2 = \frac{2\pi}{\Lambda_b}\hat{x} - (2\pi/\sqrt{3}\,\Lambda_b)\hat{y}$, and m_1 and m_2 being integers. The coherence energy for the bubble phase is

$$E_{\rm coh} = (4\pi/\sqrt{3}\,\ell^2\Lambda_b^2)\sum_{\mathbf{q}} \tilde{U}_{\rm HF}(q)\left(\frac{R}{A\ell^2 q}J_1(qR)\right)^2. \tag{10}$$

The lowest energy, shown in Fig. 2, is determined by examining bubbles with various occupancies. The stripe phase has lower energy than the Fermi sea, paired state, and the bubble crystal for the filling factors considered here.

In the above, the filling factor $\nu^* = n + 1/2$ has been viewed as half filling of CF particles on top of n filled CF-LLs. We have also carried out [19] the complementary approach in which it is modeled as half filling of CF holes on the background of $n + 1$ filled CF-LLs. The stripe phase has again been found to have the lowest energy, which gives us further confidence in the robustness of our result. It should be noted, however, that the issues regarding the stability of the unidirectional charge density wave against a modulation along the length and also against quantum fluctuations, which have been investigated in the context of electronic stripes in higher Landau levels [12], have not been considered here.

A transport anisotropy in higher electronic Landau levels, interpreted in terms of stripe formation, is observed at temperatures below $\sim$50 mK [14]. The conditions for the CF stripes are more stringent. Estimates of the critical temperature from the Hartree-Fock theory are not quantitatively reliable, but noting that the effective interaction between composite fermions at $\nu^* = n + 1/2$ is roughly an order of magnitude smaller than for electrons at $\nu = n + 1/2$, as measured by the pseudopotentials, we expect the critical temperature to also be similarly reduced. Also, the much larger period suggests the need for a high degree of density homogeneity.

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