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Composite fermion theory of excitations in the fractional quantum Hall effect

J.K. Jain[a,*], K. Park[b], M.R. Peterson[a], V.W. Scarola[b]

[a]*104 Davey Laboratory, Physics Department, The Pennsylvania State University, University Park, PA 16802, USA*
[b]*Physics Department, Condensed Matter Theory Center, University of Maryland, College Park, MD 20742, USA*

Abstract

Transport experiments are sensitive to charged 'quasiparticle' excitations of the fractional quantum Hall effect. Inelastic Raman scattering experiments have probed an amazing variety of other excitations: excitons, rotons, bi-rotons, trions, flavor altering excitons, spin waves, spin-flip excitons, and spin-flip rotons. This paper reviews the status of our theoretical understanding of these excitations.

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1. Introduction

Transport experiments told us quite early on that there is a gap to charged excitations in the fractional quantum Hall effect (FQHE) [1]. The longitudinal resistance was seen to behave like $\rho_{xx} = \exp(-\Delta/2k_{B}T)$ in a range of temperature, and the quantity Δ is interpreted as the minimum energy required to create a charged excitation. Subsequently, the excitations were probed by optical means [2–6] as well as by ballistic phonon absorption [7–9]. These and subsequent optical experiments have proved to be a treasure-trove of information on charge-neutral as well as high energy excitations not accessible in transport experiments and

have revealed a strikingly rich structure with many kinds of excitations.

There has also been substantial theoretical progress on this issue, and an excellent description of the neutral excitations has been achieved in terms of a particle hole pair of composite fermions (CFs), which dovetails nicely with our understanding of the incompressible ground states as an integral number of filled quasi-Landau levels of composite fermions. This paper will review the status of our qualitative and quantitative understanding of the excitations of the FQHE states. The reader is referred to the original articles for technical details.

The character of neutral excitations is of great interest in its own right, and their explanation is a critical test for theory. There is another motivation for their consideration. Much effort has been invested into a calculation of the gaps to charged excitations. However, a pesky factor of two discrepancy between theory and experiment has persisted

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* Corresponding author. Tel.: +1 814 863 1162; fax: +1 814 865 3604.

E-mail address: jain@phys.psu.edu (J.K. Jain).

over the years, believed to be caused by disorder for which a quantitatively reliable theoretical treatment is not available at the moment. In contrast, disorder is not likely to affect the energy of a localized neutral excitation as significantly, because of its much weaker dipolar coupling to disorder. The coupling is further diminished because the disorder in modulation-doped samples is typically smooth on the scale of the size (on the order of a magnetic length) of many spatially localized neutral excitations. We will find that the theoretical predictions, without including disorder effects, agree reasonably well with experiment.

2. The composite fermion exciton

A composite fermion [10–13] is the bound state of an electron and an even number of quantized vortices (sometimes modeled as the bound state of an electron and an even number of magnetic flux quanta, with a flux quantum defined as $\phi_0 = hc/e$). Composite fermions are to FQHE what Cooper pairs are to superconductivity. The interacting electrons at Landau level filling factor $\nu = n/(2pn \pm 1)$, n and p being integers, transform into weakly interacting composite fermions at an effective filling $\nu^* = n$. The ground state corresponds to n filled CF Landau levels (LLs), shown schematically in Fig. 1(a). The CF theory not only explains the origin of a gap at these fractional fillings, which are precisely the observed fractions, but also gives a natural insight into the excitations. Fig. 1(b) shows a typical neutral excitation, which is a particle-hole pair of composite fermions, called the CF exciton. It is analogous to the familiar exciton at integral fillings. Jain's wave functions for the CF ground state and the CF exciton are readily constructed by analogy to the known wave functions of

Fig. 1. (a) The incompressible ground state at $\nu = 2/5$, which represents two filled CF-Landau levels. (b) An exciton of composite fermions. Composite fermions are depicted as electrons carrying vortices.

the electron ground state at filling factor n, Φ_n^{gs}, and its exciton, Φ_n^{ex}:

$$\Phi_{n/(2n+1)}^{\text{gs}} = P_{\text{LLL}} \prod_{j<k} (z_j - z_k)^{2p} \Phi_n^{\text{gs}} \tag{1}$$

$$\Phi_{n/(2n+1)}^{\text{ex}} = P_{\text{LLL}} \prod_{j<k} (z_j - z_k)^{2p} \Phi_n^{\text{ex}} \tag{2}$$

where $z_j = x_j - iy_j$ is the position of the jth particle, and P_{LLL} denotes projection of the wave function into the lowest Landau level. The explicit form of these wave functions has been given in the literature [14]. The composite fermion interpretation of Φ_ν follows since, multiplication by the Jastrow factor $\prod_{j<k} (z_j - z_k)^{2p}$ is tantamount to attaching $2p$ vortices to each electron, thus, converting it into a composite fermion. These wave functions have been found to be quite accurate in tests against exact diagonalization results available for small systems [10,14]. We will not discuss the field theoretical approaches to study the excitations; the reader is referred to the articles by Lopez and Fradkin, [15] He, Simon and Halperin, [16] and Murthy and Shankar [17].

The Hamiltonian for the many electron system is given by $H = P_{\text{LLL}} \left[\frac{1}{2} \sum_{j \neq k} V(r_{jk}) \right] P_{\text{LLL}}$, where $V(r)$ is the effective two-dimensional electron–electron interaction. For a strictly two-dimensional system $V(r_{jk}) = e^2/\varepsilon r_{jk}$, where ε is the dielectric constant of the background material. A quantitative correction comes from the finite transverse extent of the electron wave function, which alters the form of the effective two-dimensional interaction at short distances. The effective interaction can be calculated once the transverse wave function is known, which in turn is determined by self-consistently solving the Schrödinger and Poisson equations, taking into account the interaction effects through the local density approximation including the exchange correlation potential [18]. The energy of the exciton,

$$\Delta^{\text{ex}} = \frac{\langle \Phi_\nu^{\text{ex}} | H | \Phi_\nu^{\text{ex}} \rangle}{\langle \Phi_\nu^{\text{ex}} | \Phi_\nu^{\text{ex}} \rangle} - \frac{\langle \Phi_\nu^{\text{gs}} | H | \Phi_\nu^{\text{gs}} \rangle}{\langle \Phi_\nu^{\text{gs}} | \Phi_\nu^{\text{gs}} \rangle} \tag{3}$$

is computed by Monte Carlo methods in the spherical geometry [19]. All results below are thermodynamic extrapolations, unless mentioned otherwise. The energies are quoted in units of $e^2/\varepsilon l_0$, where $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length.

Scarola, Park and Jain [20] determined the dispersion of the CF exciton for 1/3, 2/5, and 3/7, corresponding to one, two, and three filled CF-LLs, respectively. The dispersion, as shown in Fig. 2, is rather complicated in general, reflecting the complex density profiles of the CF-quasiparticle and the CF-quasihole. In particular, it has several minima, which are called 'rotons' (This name has its origin in analogy to superfluid ^{4}He. The minimum at $\nu = 1/3$ was earlier found by Girvin, MacDonald and Platzman [21] in a single mode approximation). It might seem that rotons would not be accessible in light scattering due to their large
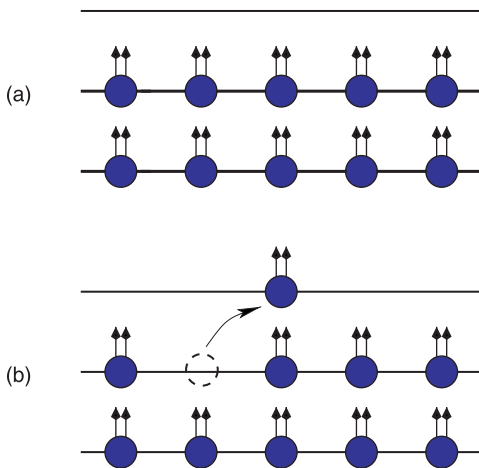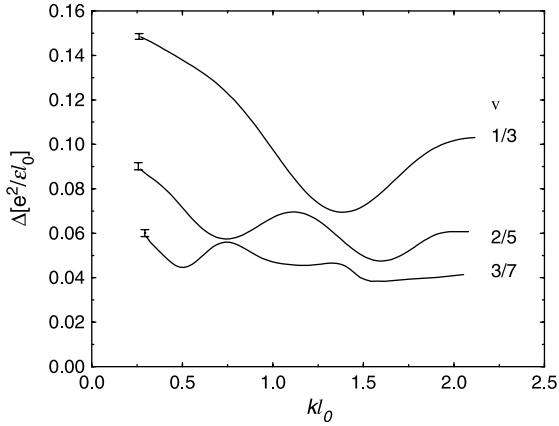
Fig. 2. The dispersions of the CF excitons at $\nu = 1/3$, 2/5, and $\nu = 3/7$ for a strictly two dimensional system (From Ref. [11]).

wave vector, but disorder-induced breakdown of wave vector conservation combined with a peak in the density of states at the roton energy makes it possible for inelastic Raman scattering to detect them. A comparison between theory and experiment is given in Table 1 [20]. For the roton, the theoretical energies, obtained with no adjustable parameters, are in excellent agreement with the observed ones. One may worry that the situation will be spoiled by Landau level mixing, but that turns out not to be the case; corrections due to LL mixing are estimated to be on the order of 5% for typical densities [20].

3. Long wave length limit: composite fermion bi-rotons

As seen in Table 1, in the small wave vector limit the calculated energy at 1/3 is off by $\sim 30\%$. It has been suggested that here the true lowest energy excitation may contain two CF-excitons [21], and there has been debate as to which excitation is being probed by Raman scattering in this case [21,22]. Further progress was hampered by the lack

of a quantitative theory of the bi-roton excitation and because the system sizes on which exact-diagonalization studies can be performed are too small to shed meaningful light on long wavelength excitations.

Park and Jain [23] constructed a wave function for the long wave length excitation by putting two rotons of opposite wave vector together, by appealing to the analogy to the two-exciton state at $\nu = n$, which contains two particle-hole pairs. In order to obtain a wave function that is orthogonal to the ground as well as the single exciton state, they consider the two-exciton wave function at angular momentum $L = 1$ in the spherical geometry [24]. The energy of this wave function is compared with the single exciton at $L = 2$, both of which correspond to $k \to 0$ in the thermodynamic limit. Fig. 3 demonstrates that the bi-roton state has 10% lower energy than the single exciton state in the long wavelength limit. Ghosh and Baskaran [25] have estimated the binding energy of the bi-roton in a variational scheme, exploiting the oriented-dipole character of the roton.

These results have relevance to Raman scattering experiments. Our theoretical understanding of the scattering cross section of the FQHE gap modes observed in Raman scattering is rather unsatisfactory, even ignoring the complications introduced by the resonant nature of Raman scattering [22,26,27]. However, as noted in the early literature [2], there is reason to expect that the bi-roton mode might couple more strongly to light in Raman scattering than the single exciton mode: the scattering cross section for the single exciton vanishes rapidly with wave vector as a result of Kohn's theorem, but there is no reason for it to vanish for the bi-roton mode. Also, while the two-exciton states form a continuum, the bi-roton states provide a peak in the density of states at the lower edge of the continuum. In order to make contact with experiment at a quantitative level, a square quantum well of width 33 nm was considered in Ref. [20], as appropriate for the experiment of Kang et al. [4] The excitation energies are further reduced due to Landau level mixing, which was

Table 1

Comparison of theory and experiment (From Ref. [20]) for the roton energy as well as the energy of the long-wavelength exciton, quoted in units of $e^2/\varepsilon l_0$

ν	$kl_0 = 0$		Roton		Reference
	Experiment	Theory	Experiment	Theory	
1/3	0.082	0.104(1)	0.044	0.050(1)	[5]
	0.084	0.113(1)	–	0.052(1)	[2]
	–	0.09(2)	0.041(2)	0.045(1)	[7]
	0.074	0.095(1)	0.047	0.047(1)	[3]
	–	0.092(1)	0.036(5)	0.045(1)	[8]
2/5	–	0.054(1)	0.021(2)	0.026(1)	[7]
	–	0.055(1)	0.025(3)	0.027(1)	[9]
3/7	–	0.044(2)	0.014(2)	0.017(2)	[7]

In Ref. [7], the roton energies were determined for 2/3, 3/5, and 4/7, which, assuming particle-hole symmetry, are the same as the roton energies at 1/3, 2/5, and 3/7, when measured in units of $e^2/\varepsilon l_0$.
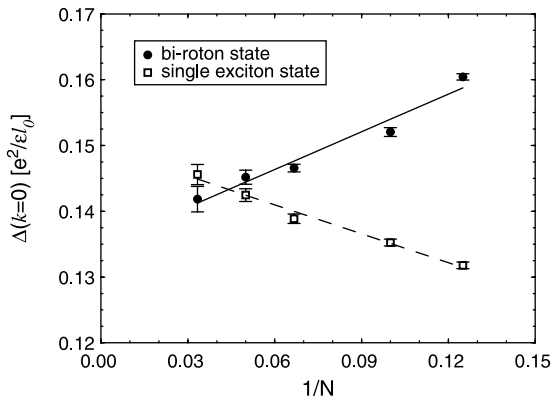
Fig. 3. The Coulomb energies as a function of $1/N$ for the single exciton and the bi-roton states in the long wavelength limit (From Ref. [23]).

estimated by Scarola et al. as a function of the density for the single roton at $\nu = 1/3$. Assuming that the percent reduction of the bi-roton energy is approximately the same (approximately 5% for typical densities) produces a realistic estimate for the energy of the bi-roton mode, plotted (dashed line) in Fig. 4 along with the experimentally determined energies [4] of the long-wavelength mode (stars). The agreement is on the level of 20% or better.

Hirjibehedin et al. [28] have investigated the wave vector dependence of the long-wavelength excitation and found the surprising result that it splits into two modes at relatively larger wave vectors. This suggests that the excitation in the zero wave vector limit is actually an unresolved doublet. The authors also estimate the binding energy of the bi-roton mode by comparing its energy with that of two unbound rotons and find it to be approximately 10% of the total bi-roton energy.
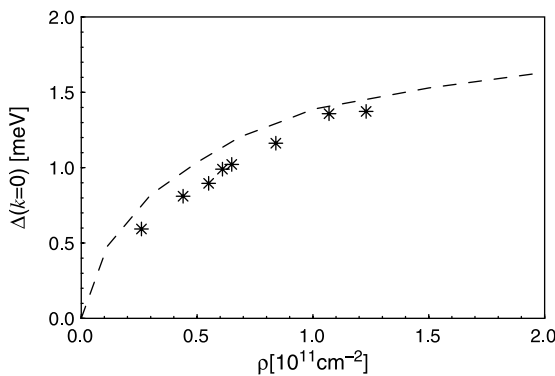

Fig. 4. Comparison between the experimental data (stars) from Kang et al. [4] and the theoretical estimation of the bi-roton bound state energy (dashed line). Theoretical estimates are obtained by considering Landau level mixing as well as finite thickness effects (From Ref. [23]).

4. Composite fermion trion: a new charged excitation

The excitation gaps at general momenta are successfully described in terms of a single CF exciton or, in the long wave length limit, a CF bi-roton. These are charge-neutral bound states of two and four composite fermions, respectively. Park [29] has considered CF excitons in the presence of an isolated CF quasiparticle or CF quasihole. Will a CF exciton take advantage of the quasiparticle or quasihole to form a new bound state involving three composite fermions? Such a charged complex is known as a trion.

There has been considerable interest in the trion state of electrons in two spatial dimension because of the increased binding energy due to the reduced dimensionality [30–32]. Trion excitations in strong magnetic fields have been studied by a number of authors; all of whose works, however, have concentrated on trion bound states of electrons in the conduction band and holes in the valence band [33–36]. On the contrary, the trion state in this section is formed by CF-quasiparticles and CF-quasiholes which reside in the same layer, band and electronic LL.

Park [29] used a simple criterion for the stability of the CF trion: can the CF exciton lower its energy by forming a bound state with an already present quasiparticle? The energy cost is computed as a function of the momentum of the constituent neutral CF exciton, $k_{ex}l_0$, which can be roughly used as a measure of CF trion size. A subtle, but important technical point in constructing the wavefunction for CF trion is that it is not straightforward to increase electron numbers while staying at a fixed value of $k_{ex}l_0$: because of the nature of the spherical geometry, $k_{ex}l_0$ is also
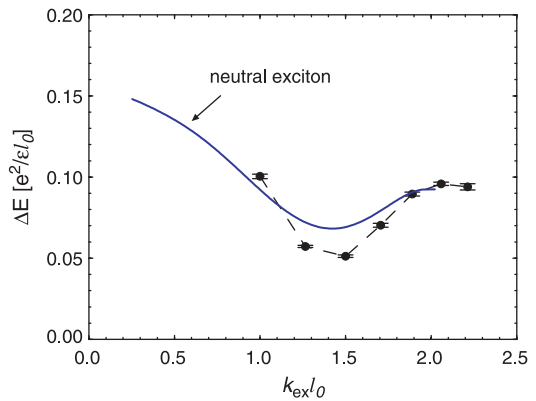

Fig. 5. Energy of the a CF exciton in the presence of a solitary CF quasiparticle (dashed line) as a function of the momentum ($k_{ex}l_0$). The quantity $k_{ex}l_0$ is proportional to the distance between the CF-quasiparticle and the CF-quasihole of the constituent neutral exciton, and can, therefore, be used as a measure of the size of CF trion itself. The solid line shows the energy of a neutral CF exciton. The fact that the former is lower near $k_{ex}l_0 = 1.5$ shows that the CF-exciton forms a bound state with the CF-quasiparticle to form a trion (From Ref. [29]).

changed upon increasing the system size in order to obtain a unique trion state. The method in Ref. [29] is a particular way of scanning CF trion states. Exact diagonalization studies for the trion are not easy to carry out in practice.

Fig. 5 shows the Coulomb energy of a CF exciton in the presence of a CF-quasiparticle, as a function of $k_{ex}l_0$, which is compared with that of a neutral CF exciton. The former has a lower energy in the vicinity of $k_{ex}l_0 = 1.5$, demonstrating that the CF exciton and the CF-quasiparticle form a trion bound state. The binding energy of the trion is estimated to be roughly $0.02 e^2/\varepsilon l_0$. As expected, there is no energy gain for large $k_{ex}l_0$, because here the CF trion represents a far-separated, independent collection of quasiparticles and quasiholes.

The CF-trions ought to be observable in resonant inelastic light scattering experiments. The conventional interpretation of the excitations measured in those experiments is in terms of neutral excitons. However, since, there presumably are always some localized quasiparticles in real experiments (induced by disorder), it is expected that light-scattering will also excite CF trions. Interestingly, Hirjibe-hedin et al. [28] have seen additional excitations at energies slightly below the ordinary roton energy, which may be just the CF trion excitations discussed above. They lie 0.005–0.01 $e^2/\varepsilon l_0$ below the roton, which compares favorably with the theoretical estimate of the binding energy of the CF-trion (The theoretical estimates, not including finite thickness and disorder effects, are known to overestimate various energy gaps).

5. Composite fermion flavor altering excitations

The work presented in this section was motivated by the light scattering experiment by Hirjibehedin et al. [37], who investigated the filling factor range $1/3 \geq \nu \geq 1/5$. The relevant composite fermions carry four vortices (^{4}CFs) in this filling factor range. The experimental results show that while new low-energy excitations appear for $\nu < 1/3$, the excitations from $\nu > 1/3$ do not disappear but evolve continuously as the filling factor changes across $\nu = 1/3$ toward $\nu \to 1/5$. At first sight, that appears inconsistent with the understanding that the relevant composite fermions in the filling factor region $\nu < 1/3$ are different from those in the region $\nu > 1/3$.

Peterson and Jain [38] have proposed that these are a new class of excitations. The previous sections have described excitations for which the integrity of the composite fermion remains intact, i.e. the 'flavor' of the CF does not change (The composite fermions carrying different numbers of vortices are said to have different flavors. They are represented by ^{2p}CFs.) These new excitations correspond to situations for which some of the $2p$ vortices attached to a ^{2p}CF are stripped away. These are analogous to the pair breaking excitations in a superconductor.
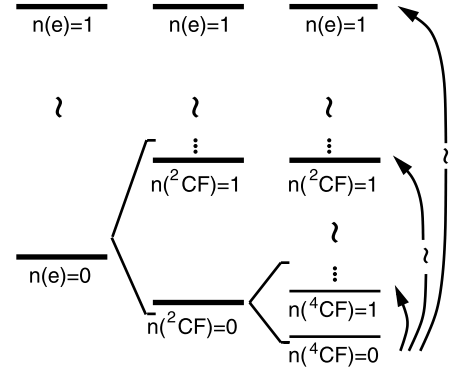
The basic idea is explained in Fig. 6. The lowest

Fig. 6. Schematic view of how the lowest electronic Landau level splits into CF-LLs of composite fermions carrying two vortices (^{2}CFs), and how the lowest ^{2}CF-LL further splits into ^{4}CF-LLs. The Landau level index is denoted by n(P) where 'P' is the type of relevant particle; 'e' (electron), '^{2}CF,' or '^{4}CF.' The three ladders of flavor changing excitations are indicated in the rightmost column (From Ref. [38].).

electronic LL splits into CF-Landau levels of ^{2}CFs, as depicted in the middle column of Fig. 6 The intra-LL excitations of electrons are described as inter-CF-LL excitations of ^{2}CFs. However, the inter-electronic LL excitation, or Kohn mode, is still supported by the system. This simple case shows the coexistence of excitation modes at two different energy scales: the low energy $\Delta^{2 \to 2}(k)$ excitations conserve the CF flavor, whereas the high energy $\Delta^{2 \to 0}(k)$ excitations involve a transformation of a ^{2}CF into a ^{0}CF (electron). For $\nu < 1/3$ the situation is analogous. Here the lowest ^{2}CF-LL undergoes a further splitting into ^{4}CF-LLs, as shown in the rightmost column of Fig. 6. Although we have only ^{4}CFs in the ground state, there are now three ladders of excitation: (i) $\Delta^{4 \to 4}(k)$ flavor conserving excitations, which were not present for $\nu > 1/3$; (ii) $\Delta^{4 \to 2}(k)$ excitations involving a change of ^{4}CFs into ^{2}CFs, which are a continuous evolution of $\Delta^{2 \to 2}(k)$ excitations for $\nu > 1/3$; and (iii) $\Delta^{4 \to 0}(k)$, which are similar to $\Delta^{2 \to 0}(k)$ and $\Delta^{0 \to 0}(k)$. The flavor changing excitations correspond to a partial ionization of the composite fermion. This interpretation is supported by the earlier calculations of Wójs and Quinn [39], who show that the bands in the energy spectrum (obtained in exact diagonalization studies) can be characterized by the number of vortices bound to electrons.

To make contact with experiment, it is convenient to focus on the zero wave vector mode and the roton, which will be labeled $\Delta(0)$ and $\Delta(R)$, respectively. Fig. 7 shows the energies $\Delta^{4 \to 4}(0)$, $\Delta^{4 \to 4}(R)$, and their $4 \to 2$ counterparts for a six particle system for zero thickness as well for a quantum-well sample of width 33 nm. While the $4 \to 2$ modes evolve continuously out of the $2 \to 2$ modes at $\nu \geq 1/3$, the $4 \to 4$ modes are new. The calculated values for $\Delta^{4 \to 4}(R)$ are approximately ~ 0.2 meV (using parameters of the experiment in Ref. [37]), decreasing a little bit with increasing magnetic field. The experimental values go from
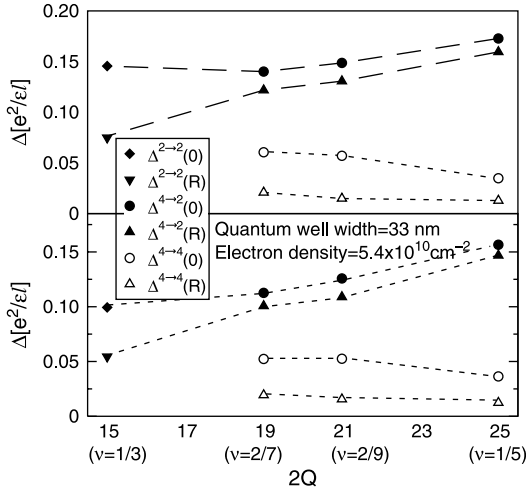
Fig. 7. Excitation energies $\Delta^{2p \rightarrow 2p'}$ of the zero wave vector and roton modes are shown as a function of magnetic field (2Q denotes the number of flux quanta penetrating the sample). The upper panel gives the energies for strictly two-dimensions, whereas the lower panel incorporates corrections due to finite thickness using parameters from the experimental sample of Ref. [37] (quantum well width=33 nm and density $=5.4 \times 10^{11}$ cm^{-2}). All results are for N=6 particles (From Ref. [38]). The dashed lines are a guide to the eye.

~ 0.2 to ~ 0.11 meV at $\nu=1/5$, decreasing with magnetic field. For the $\Delta^{4 \rightarrow 2}(R)$ mode, the theoretical value starts at approximately ~ 0.6 meV and increases to ~ 2.0 meV as a function of magnetic field strength. The experimental value starts at ~ 0.45 meV and goes on increasing with magnetic field to ~ 0.7 meV. The theory is in qualitative agreement with experiment, both in regard to the continuity of some modes across $\nu=1/3$ and appearance of new modes for $\nu < 1/3$, and the observed filling factor dependence. Considering the small size of the numerical system, we find the level of quantitative agreement to be adequate.

6. Spin-flip excitons and spin-flip rotons

The excitations involving spin reversal are also of interest. They are identified experimentally by ascertaining the dependence of their energy on the magnetic field: the energy of a spin reversed mode contains a term proportional to B in addition to the usual $\sqrt{B_\perp}$ dependent term arising from interactions. One example of such an excitation is the spin-wave mode [2,3]; its energy in the small wave vector limit is equal to the Zeeman splitting, denoted by E_Z. Recently, substantial progress has been made in extending the experiments to other fractions, e.g., $\nu=2/5$ and 3/7, where a much richer structure has emerged. In particular, Kang et al. [6] have reported observation of spin-reversed modes other than the spin wave at these fractions. At $\nu=2/5$, they find a mode with energy approximately equal to $2E_Z$;

the near absence of $\sqrt{B_\perp}$ dependence of the energy indicates that it is not modified by the Coulomb interaction. Another striking observation was of a mode at $\nu=3/7$ which has an energy smaller than E_Z, roughly 0.4 E_Z.

At $\nu=1/3$, which maps into $\nu^*=1$ of composite fermions, the lowest energy spin reversed mode is the one in which the composite fermion flips its spin while remaining within the lowest CF-LL. Nakajima and Aoki [40] showed, by comparison with exact diagonalization results [41], that the CF theory provides an excellent description of the spin wave mode at $\nu=1/3$. The possibility of several kinds of modes is immediately obvious in the CF theory. Consider, for example, $\nu=2/5$, where the up-spin states of the CF-LLs, labeled 0 and 1, are fully occupied. Three possible low energy excitations are: (i) $1\uparrow \rightarrow 0\downarrow$, (ii) $1\uparrow \rightarrow 1\downarrow$, and (iii) $0\uparrow \rightarrow 0\downarrow$. The last two conserve the CF-LL index, whereas the composite fermion lowers its LL index in the first. Mandal and Jain [42] evaluated the energy dispersions of the spin-reversed excitations numerically using the CF theory. (The Coulomb energy of the exciton measured relative to the ground state is denoted by Δ_c^{ex}; the Zeeman energy $E_Z=|g|\mu_B B$ must be added to it to obtain the full energy of the spin-reversed excitation.) The three Coulomb eigenvalues obtained in this way are shown in Fig. 8, labeled (a), (b), and (c). For $\nu=3/7$, the Coulomb Hamiltonian is diagonalized in the subspace defined by six modes: $2\uparrow \rightarrow 0\downarrow$; $2\uparrow \rightarrow 1\downarrow$; $1\uparrow \rightarrow 0\downarrow$; $2\uparrow \rightarrow 2\downarrow$; $1\uparrow \rightarrow 1\downarrow$; and $0\uparrow \rightarrow 0\downarrow$. The resulting spectrum is shown in Fig. 9.

The familiar spin-wave mode is recovered at small wave vectors, as expected. The principal result of the calculations is that at finite wave vectors, the lowest energy mode is not the spin wave, but an excitation that involves a spin-flip associated with a CF-LL transition. Further, this excitation has a roton minimum, termed the 'spin-flip roton' (See also Murthy [17].). In the 2D limit, the energies of the spin-flip
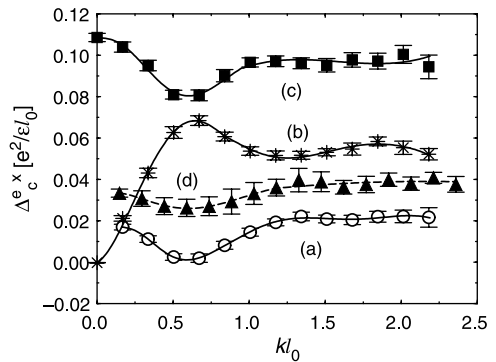

Fig. 8. The curves labeled (a), (b), and (c) show the dispersions of three spin reversed modes for $N=30$ particles for the Coulomb potential $V(r)=e^2/\varepsilon r$. The ground state is assumed to be fully polarized. The error bars indicate the estimated statistical error in Monte Carlo. The curve (d) shows the energy of the spin reversed excitation $0\downarrow \rightarrow 1\uparrow$ of the unpolarized 2/5 state for $N=38$ (From Ref. [42]).
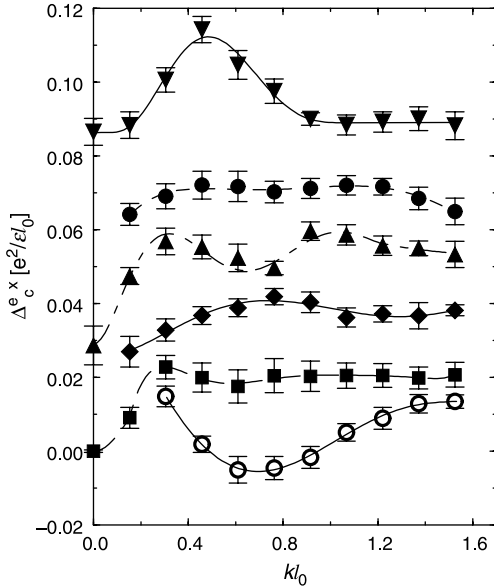
Fig. 9. The spin reversed excitations of fully polarized 3/7 state. The results are for the Coulomb potential $V(r)=e^2/\varepsilon r$ for $N=39$ particles (From Ref. [42]).

rotons at $\nu=2/5$ (for the $1\uparrow\rightarrow0\downarrow$ mode) and $\nu=3/7$ ($2\uparrow\rightarrow0\downarrow$) are estimated to be $-0.0024(18)e^2/\varepsilon l_0$ and $-0.0091(36)e^2/\varepsilon l_0$, respectively. At $\nu=2/5$, the interaction energy of the spin-flip roton is very close to zero, implying that its total energy is close to E_Z. At $\nu=3/7$, the interaction energy of the roton is negative, which, for typical parameters, leads to a total energy of approximately 0.5 E_Z. Mandal and Jain associated the spin-roton with the low-energy excitation observed by Kang et al. at this filling factor [6] both because no other such low energy mode is known, and because the calculated energy of the spin-flip roton is in reasonable agreement with the observed energy.

Dujovne et al. [43] have studied spin reversed excitations in the filling factor range $2/5\geq\nu\geq1/3$ by resonant light scattering and analyzed their results successfully in terms of transitions across spin-split Landau levels of composite fermions. Their results agree well with the dispersion of the spin-flip mode at $\nu=2/5$ computed in Ref. [42] in which both the spin orientation and the CF-Landau level index change simultaneously (i.e. the $1\uparrow\rightarrow0\downarrow$ mode). Dujovne et al. also observed an abrupt change in the energies of the spin-flip modes at an intermediate filling, which indicates a not-yet-understood qualitative change in the nature of the state of the spin-reversed composite fermions. In a more recent paper, Dujovne et al. [44] have found that the energy of the composite-fermion spin-flip excitation collapses as $\nu\rightarrow1/2$, which they link to a partial polarization of the CF Fermi sea; this interpretation is confirmed by the absence of such a collapse when the Zeeman energy is sufficiently enhanced by the application of an additional parallel magnetic field (tilted field experiment).

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