Pseudospin Quantum Computation in Semiconductor Nanostructures

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(Received 9 April 2003; revised manuscript received 21 July 2003; published 16 October 2003)

We theoretically show that spontaneously interlayer-coherent bilayer quantum Hall droplets should allow robust and fault-tolerant pseudospin quantum computation in semiconductor nanostructures with voltage-tuned external gates providing qubit control and a quantum Ising Hamiltonian providing qubit entanglement. Using a spin-boson model, we estimate decoherence to be small ($\sim 10^{-5}$).

DOI: 10.1103/PhysRevLett.91.167903 PACS numbers: 03.67.Pp, 03.67.Lx, 73.21.La, 73.43.–f

Among the stringent requirements for viable quantum computer architectures are robust (i.e., relatively decoherence-free) and scalable qubits (i.e., quantum two-level systems) allowing single- and two-qubit operations necessary for quantum computation [1]. The scalability requirement makes semiconductor-nanostructure-based quantum computer architectures particularly attractive, and two spin-based semiconductor quantum computer architectures, one using electron spins in GaAs quantum dots [2] and the other using Si donor spin states [3], have attracted considerable attention. The proposed advantage of solid state spin quantum computation over the corresponding charge or orbital state quantum computation is the long decoherence time for spin states (μs or longer at low temperatures) compared with orbital states (*ps* or less) allowing, at least in principle, robust quantum computation using spin qubits in semiconductor nanostructures. A very serious problem in solid state spin quantum computation is, however, the measurement of single spin states crucial for the quantum computation readout. (There is no known solid state experimental technique for measuring a single spin, i.e., one Bohr magneton, and a great deal of experimental activity is currently being focused on measuring a single spin in semiconductor structures.) In this Letter, we theoretically establish the practical possibility of a novel pseudospin quantum computation in semiconductor nanostructures which synergetically combines the robustness of spins (i.e., long decoherence time) with the ease of qubit-specific measurement of charge states by using mesoscopic ''coherent'' charge states in quantum Hall droplets [4,5].

Quantum Hall systems offer well-studied mesoscopic quantum states with the potential for dynamic manipulation with long dephasing times. Surprisingly, little work has gone into exploring the possibility of engineering quantum Hall states for the purpose of quantum computation. Mozyrsky *et al.* [6] have explored the possibility of using nuclear spins as qubits with an interaction mediated by a two-dimensional electron gas in the quantum Hall regime. Recently, Yang *et al.* [4] have made a proposal for a quantum Hall two-level system using the charge degrees of freedom in two vertically coupled quantum dots in a large magnetic field, a system which is currently the subject of intense experimental study [7]. In these systems, the layer degree of freedom acts as a pseudospin, controllable through external gates. The incompressibility of the finite size quantum Hall liquid preserves the integrity of the two-level system while the mapping between layer index and pseudospin relies on the presence of spontaneous interlayer phase coherence [8]. Drawing upon the direct analogy between number fluctuations in the Cooper pair box experiment [9] and fluctuations in the layer degree of freedom in bilayer quantum Hall droplets (BQHDs), an even-odd effect in the Coulomb blockade spectra of BQHDs has been proposed [5] as a simple measure of spontaneous interlayer phase coherence (and, hence, the robustness) of the two-level system discussed in Ref. [5].

The remainder of this article will be concerned with entangling two BQHDs which, when isolated, demonstrate the even-odd effect independently. Establishing controllable entanglement is crucial to performing large scale quantum computing. Our primary result is that the Coulomb interaction offers a natural entangling mechanism, opening the possibility of large scale quantum computing using BQHDs. We find that, for weak coupling, the Coulomb interaction between two laterally separated BQHDs can be mapped onto a quantum Ising model with a tunable, effective magnetic field. This two-qubit Hamiltonian allows for relatively simple implementation of a controlled-NOT operation [10] which, when combined with single qubit operations, provides a universal set of quantum gates [11]. We further address the extent to which phonons and voltage fluctuations in the leads dephase our system.

We begin by considering a set of two parabolically confined quantum dots vertically separated by a distance d under a transverse magnetic field, B. The two dots will form a BQHD for appropriate magnetic fields and layer spacings. We further assume there to be a small, odd number of electrons distributed between the two droplets. In a large magnetic field, the Coulomb interaction exhibits energy cusps at configurations corresponding to bulk, bilayer quantum Hall states. We focus our attention on the

maximum density droplet (MDD) which is the mesoscopic realization of the bilayer quantum Hall state at total Landau level filling $\nu_T = 1$.

The Hamiltonian for an isolated BQHD in the Fock-Darwin basis is

$$H = H_0 + \hat{P} V_{\text{coul}} \hat{P}, \tag{1}$$

where $H_0 = \frac{1}{2}(\sqrt{\omega_c^2 + 4\omega_0^2} - \omega_c)\hat{L}_z$, with $\hat{L}_z$ being the total angular momentum in the z direction. Also, ω_c is the cyclotron frequency and ω_0 parametrizes the parabolic confining potential. $\hat{P}$ is the lowest Landau level (LLL) projection operator and V_{coul} represents the usual Coulomb interaction between electrons:

$$\frac{V_{\text{coul}}}{e^2/\varepsilon a} = \sum_{i<j\in\uparrow} \frac{1}{r_{ij}} + \sum_{k<l\in\downarrow} \frac{1}{r_{kl}} + \sum_{i\in\uparrow,k\in\downarrow} \frac{1}{\sqrt{r_{ik}^2 + (\frac{d}{a})^2}}, \tag{2}$$

where ε is the GaAs dielectric constant, and r_{ij} is the lateral separation between the ith and the jth electron. The natural unit of length is the modified magnetic length $a = l_B(1 + 4\omega_0^2/\omega_c^2)^{-1/4}$ which reduces to the planar magnetic length, $l_B = \sqrt{\hbar c/eB}$, when the cyclotron energy is much larger than the confining potential energy. In the above, we have used a pseudospin representation to describe the double layer system: $\uparrow$ and $\downarrow$ distinguish different layers. In general, we define the pseudospin operator:

$$\boldsymbol{S} \equiv \frac{1}{2}\sum_m c_a^\dagger(m)\sigma_{ab}c_b(m), \tag{3}$$

where $c_a^\dagger(c_b)$ creates (annihilates) an electron in the layer $a(b)$ with single-particle angular momentum m. $\boldsymbol{\sigma}$ are the usual Pauli matrices. $\hat{S}_z$ measures half the electron number difference between layers, and $\hat{S}_x$ is associated with interlayer tunneling. We take the real spin to be fully polarized either because of the large Zeeman coupling or because of electron-electron repulsion, i.e., Hund's rule.

We diagonalize H in the basis of LLL single-particle eigenstates. In particular, we focus our attention on the part of the Hilbert space containing the MDD state. It was shown in Ref. [5] that, in the absence of tunneling, the two degenerate states with $S_z = \pm\frac{1}{2}$ are separated from states with different S_z by a large charging energy. These two states, labeled $|\uparrow\rangle$ and $|\downarrow\rangle$, form the qubit basis of our two-level pseudospin system.

Now consider a second BQHD along the x axis at a distance R away from the first BQHD, as shown in Fig. 1. To avoid lateral tunneling, we keep the distance between BQHDs larger than the MDD diameter which is the diameter of the largest Fock-Darwin orbital, roughly $2\sqrt{N}a$, where N is the total number of electrons in both BQHDs. To include the inter-BQHD Coulomb interaction, we first note that for $R \gg a$ the low lying energy levels of the two BQHDs contain a set of four degenerate product states. For $R \sim 8a$, the Coulomb interaction between electrons in different BQHDs will favor the two pseudospin unpolarized states. We will verify, by direct calculation, that the inter-BQHD interaction energy can be made smaller than the intra-BQHD energy gap. We will consider a regime where the inter-BQHD interaction is too weak to produce intra-BQHD excitations, leaving the density unperturbed. Then, to a first approximation, we may take the basis states of the weakly interacting system of two BQHDs to be product states.

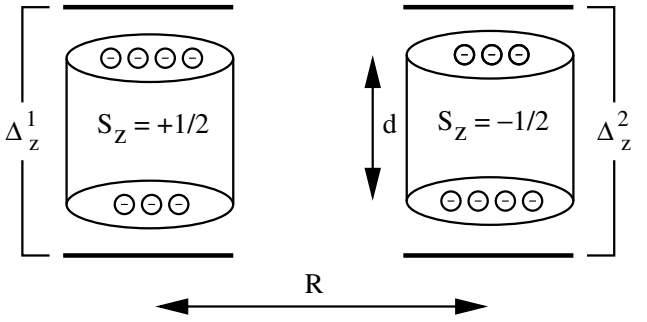

FIG. 1. Schematic representation of two bilayer quantum Hall droplets separated by a center-to-center distance R. Individual droplets are vertically separated by a distance d. The left set of droplets has one extra electron in the top layer giving it a net pseudospin, $S_z = +1/2$. The right set of droplets has pseudospin $S_z = -1/2$. This configuration corresponds to the basis state $|\uparrow\downarrow\rangle$. Δ_z^1 and Δ_z^2 are the relative bias voltages between the layers in the left and right set of droplets, respectively.

We now calculate the inter-BQHD interaction matrix between the four product states: $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$. First note that the Coulomb interaction does not flip pseudospin so that all off-diagonal matrix elements vanish. The four diagonal matrix elements do not vanish. The inter-BQHD Coulomb interaction, within our restricted Hilbert space, therefore maps onto an Ising interaction:

$$H_I = \frac{J}{2}\sigma_z^1\sigma_z^2, \tag{4}$$

where we define the exchange splitting to be

$$J = \langle\uparrow\uparrow| \sum_{i,j} V(R, d; \mathbf{r}_i, \mathbf{r}_j')\, |\uparrow\uparrow\rangle - \langle\uparrow\downarrow| \sum_{i,j} V(R, d; \mathbf{r}_i, \mathbf{r}_j')\, |\uparrow\downarrow\rangle, \tag{5}$$

where

$$V(R, d; \mathbf{r}_i, \mathbf{r}_j') = \frac{e^2}{\varepsilon' a} \frac{1}{\sqrt{(x_i - x_j' + R/a)^2 + (y_i - y_j')^2 + (d/a)^2}}. \tag{6}$$

$\mathbf{r}$ ($\mathbf{r}'$) indicates the radial vector in the x-y plane in the left (right) BQHD and ε' is the inter-BQHD dielectric constant.

Note that with this definition the Coulomb interaction will favor an antiferromagnetic interaction with $J > 0$. The states $|\uparrow\rangle$ and $|\downarrow\rangle$ are nontrivial, many-body eigenstates of H.

Figure 2 is a log-log plot of J as a function of the inter-BQHD separation R for a total of 6, 10, and 14 electrons distributed between two BQHDs with vertical spacing $d = a$. For $R \sim 8a$, we find that J is appropriately smaller than the intra-BQHD edge excitation gap which is $\sim 0.04[e^2/(\varepsilon a)]$ [5]. At sufficiently large distances, an odd number of electrons in a single BQHD in the MDD state can be thought of as a dipole. Figure 2 shows that $J \propto R^{-3}$ for $R \gtrsim 25a$, demonstrating that the inter-BQHD interaction is dipolar *only* for sufficiently large distances. In the regime of interest $R \sim 8a$, J decreases much faster than R^{-3}. We find that for $R \sim 20a$ the exchange splitting is already as low as $J \sim 0.5\ \mu$eV at $B = 9$ T. This suggests that the interaction between a collection of BQHDs will be effectively nearest neighbor given that we have not considered effects such as finite layer thickness and, in general, screening of the inter-BQHD interaction, which significantly reduce the strength of the Coulomb interaction.

We now allow for interlayer tunneling within a single BQHD. The tunneling Hamiltonian can be written in terms of the pseudospin operator:

$$H_t = -t\hat{S}_x, \tag{7}$$

where t is the single-particle, interlayer tunneling gap. In the reduced Hilbert space $|\uparrow\rangle$ and $|\downarrow\rangle$ of the ith BQHD, we find [4,5]

$$H^i_{\text{red}} = -\Delta^i_x \sigma^i_x + \Delta^i_z \sigma^i_z. \tag{8}$$

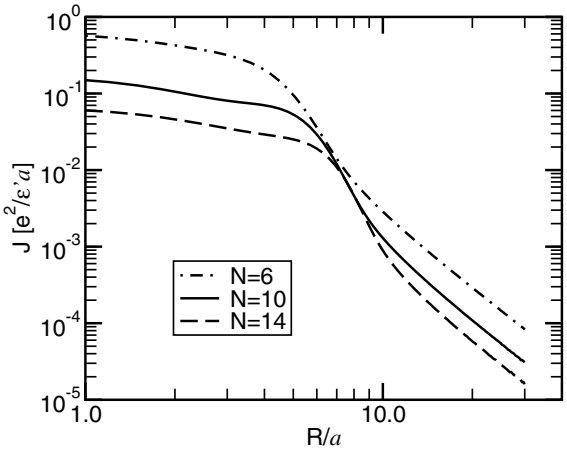

FIG. 2. The exchange splitting, J, between the pseudospin singlet and triplet states of a pair of bilayer quantum Hall droplets versus lateral separation R. J is evaluated for systems with a total of 6, 10, and 14 electrons. The vertical separation between droplets is $d = a$. The droplet diameter is below $7.5a$ for each curve. ε' is the dielectric constant between bilayer quantum Hall droplets.

In the limit of a small single-particle tunneling gap t, $\Delta_x = t\langle\uparrow|\,\hat{S}_x\,|\downarrow\rangle$. Also, Δ_z is the relative bias voltage between layers. Δ_x is the *renormalized tunneling gap* which is greatly enhanced from the single-particle tunneling gap, t, by the Coulomb interaction. H_{red} acts as the Hamiltonian of an effective magnetic field pointing in the x-z plane. The effective field will reorient the direction of the on-site pseudospin.

The system discussed here has the advantage of being scalable. One may consider a large number of BQHDs coupled via nearest neighbor interactions. Two examples include a linear chain of closely spaced BQHDs or a planar, triangular lattice. The reduced Hamiltonian of a weakly coupled, many-BQHD system corresponds to a quantum Ising model:

$$H_{\text{total}} = \sum_i [-\Delta^i_x \sigma^i_x + \Delta^i_z \sigma^i_z] + \sum_{i,j} \frac{J^{ij}}{2} \sigma^i_z \sigma^j_z. \tag{9}$$

To perform a quantum logic operation, the single qubit parameters in H_{total} should be tunable. First, Δ_z may be adjusted by applying a gating bias to each BQHD. Δ_x can be tuned by changing t through an in-plane magnetic field or a gating mechanism which alters the lateral position of the dots. The interqubit parameter, J, may be tuned by placing a third BQHD between the original two BQHDs. The inter-BQHD interaction can be turned on and off by placing an even or odd number of electrons in the intermediate BQHD or by depleting it completely. In fact, it is not necessary to physically change J because it may be possible to effectively tune the qubit coupling through a series of refocusing pulses. This technique has been used to implement quantum algorithms in NMR liquids governed by H_{total}, where the fixed coupling is between nuclear spins. It is also important to note that the architecture proposed here has the additional advantage of being charge based, simplifying readout. Single electron transistors, in principle, already have the capability [10,12] of measuring the charge imbalance between states with $S_z = +1/2$ and $S_z = -1/2$.

Finally, we consider the important issue of qubit robustness by showing long pseudospin coherence times in BQHD systems. We consider two sources of dephasing in a single BQHD: phonons and voltage fluctuations in the leads. Phonons readily couple to single electron degrees of freedom in quantum dots. This may potentially destroy our proposed two-level system through leakage to excited states. We note, however, that the rigidity of the incompressible quantum Hall droplet lifts the large degeneracy of the excited states, thereby suppressing phonon induced excitations. To show this quantitatively, consider the following general Hamiltonian for electron-phonon coupling:

$$H_{\text{e-p}} = \sum_{\boldsymbol{k}} M_{\boldsymbol{k}} \rho(\boldsymbol{k})(a_{\boldsymbol{k}} + a^\dagger_{-\boldsymbol{k}}), \tag{10}$$

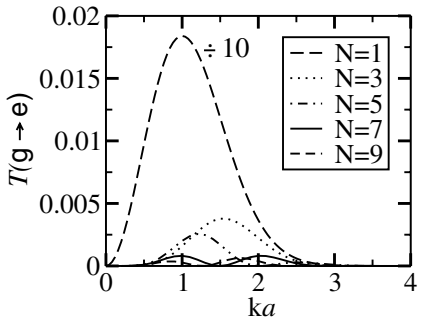

FIG. 3. The probability, T, that a perturbation of the electron density will excite a bilayer quantum Hall droplet from the ground, maximum density droplet state to an edge state as a function of wave vector, k. The vertical separation between droplets is $d = a$. The particle number, N, is increased from 1 to 9, showing a dramatic decrease in T.

where M is an arbitrary electron-phonon interaction matrix element and $a_{\boldsymbol{k}}$ and $a_{\boldsymbol{k}}^{\dagger}$ annihilate and create phonons of wave vector $\boldsymbol{k}$ in the x-y plane. $\rho(\boldsymbol{k})$ is the density operator, given by

$$\rho(\boldsymbol{k}) = \sum_{j=1}^{N} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j}. \tag{11}$$

We use first order perturbation theory to estimate the change in the rate at which phonons couple to our two-level system as we increase the system size, in which case the electron-phonon scattering rate is proportional to the transition matrix element between the initial and final electronic states. We calculate the transition matrix element between our proposed two-level ground state, g, and the lowest excitation, e:

$$T(g \to e) = |\langle e|\rho(\boldsymbol{k})|g\rangle|^2. \tag{12}$$

T measures the probability that a phonon of wave vector $\boldsymbol{k}$ will induce an excitation from the ground state to the excited state. The ground and excited states are computed from H using exact diagonalization at angular momenta corresponding to the MDD and its edge excitation, respectively [5]. Figure 3 plots the matrix element T as a function of $|\boldsymbol{k}|$ for several different particle numbers. From the plot, we see that phonon coupling to edge modes is suppressed as we increase the system size suggesting that the incompressible system studied here will be less sensitive to dephasing from phonons than analogous systems utilizing single-particle, charge degrees of freedom.

Another primary mechanism coupling pseudospin to the environment is similar to the corresponding Cooper pair box problem [10], through voltage fluctuations, δV, (e.g., in the gating potential) leading to the standard spin-boson model for decoherence of a single qubit:

$$H_{\mathrm{SB}} = -\Delta_x\sigma_x + \Delta_z\sigma_z + \gamma e\delta V\sigma_z, \tag{13}$$

where γ is a dimensionless parameter related to the qubit and gate capacitances. Following standard spin-boson techniques [10], the voltage fluctuations due to an external impedance with Ohmic dissipation can be modeled by a harmonic oscillator bath, leading to an equivalent mapping of our pseudospin decoherence problem to the corresponding Cooper pair box decoherence [9,10] problem. This then leads, after some straightforward algebra, to the following low temperature estimate for the decoherence factor α, which is the ratio of the dephasing rate to the elementary logic operation rate:

$$\alpha = \gamma^2\frac{4R_v}{R_K} = \left[\frac{C_g}{2C_g + C_d}\right]^2\frac{4R_v}{R_K}, \tag{14}$$

where C_g and C_d are the gate and quantum dot capacitances, respectively; $R_v(\sim 50\ \Omega)$ is the typical impedance of the voltage circuits, and $R_K = h/e^2$. Using results from Ref. [5] to estimate the dot capacitance and reasonable values for the gate capacitance, we get $\alpha \sim 10^{-5}$, establishing that robust fault-tolerant quantum computation should be possible in pseudospin quantum Hall systems. Although voltage fluctuations are likely to be the dominant decoherence mechanism on our proposed BQHD qubit, there are other possible dephasing channels which should be considered in the future. In particular, we suggest that the time scale of $1/f$ noise associated with charge fluctuations [13] is long enough to be dealt with using refocusing.

We acknowledge helpful conversations with J. K. Jain. This work was supported by ARDA.

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